FOUNDATIONS OF TREE RISK ANALYSIS:
Use of the $t/R$ ratio to Evaluate Trunk Failure Potential

By Jerry Bond

Trunks with cavities and large decay pockets were the first tree defects to receive careful, quantitative analysis for harm potential and have played a predominant role in tree risk analysis ever since.

By the mid-1990s, prudent risk analysis of these defects seemed well established with the publication of seemingly clear mathematical guidelines. But recent studies have undermined the very basis of those guidelines, raising serious difficulties for practitioners trying to use them. In this article, I want to examine the background and current status of this important topic and then outline its practical implications.

Background

Excellent reviews exist of trunk failure hazard assessment techniques (Lonsdale 2003) and strength-loss formulas (Kane et al. 2001) used to evaluate the failure potential of cavities or large decay pockets. As Kane et al. demonstrated, most of the proposed formulas use a ratio of cubed diameters: the inside diameter ($d$) of the defect divided by the outside diameter ($D$) of the tree (ignoring the bark). This ratio is usually represented as $d^3/D^3$.

The origin of this formula lies in engineering, where the resistance to bending of a standing pipe-shaped object, known as the “second moment of area” is calculated using the ratio of those diameters to the fourth power (Niklas 1992).

Willis Wagener undertook the reduction to the third power in his groundbreaking research paper of 1963. His intent was to produce more conservative estimates than those used by earlier researchers and to account for differences between ideal pipes, which are perfectly round and homogeneous, and real-world trees with their broad variation of material, geometry, and architecture. As Wagener explained,

Field experience indicates that a conifer can suffer up to one-third loss in strength—equivalent to approximately a 70 percent loss in total wood diameter inside bark—without materially affecting the safety of a tree if the weakening defect is heart rot uncomplicated by other defects.

Wagener specifies conifers in this passage because he sees limited application of his quantitative approach to hardwoods:

A specific standard of loss in strength, such as one-third, is less applicable to hardwoods because of [these] features . . . : (a) the difference in basic form between hardwoods and conifers, (b) the strong and variant influence of leverage on breakage potential, (c) the high mechanical strength of the wood of many hardwood species, and (d) the fact that trunk failures are relatively rare in occurrence except in weak-wooded species such as poplars.

The most widely adopted quantitative evaluation method in America is the one introduced by Mattheck and Breloer (1994). As they wrote in their concluding practical guide: “To exclude the possibility of failure from cross-sectional flattening it is therefore sufficient to fulfill the requirement $t/R > 0.3$ to 0.35 for trees with full crowns,” where $t$ is the radial thickness of sound wood and $R$ the radius of the stem. (The term “cross-sectional flattening” refers here to one type of hollow column failure that comes from local, or Brazier, buckling (sometimes called “hose pipe kinking”) in which a circular form flattens and then separates into individual plates that fail. See Kane et al. 2001). Furthermore, the authors argued that neither trunk size nor wood strength plays a role: large or small trees, strong oaks or weak willows—all adhered to the same requirement.

The authors made one important qualification that has not received as much attention as it deserved. Lower $t/R$ values could be tolerated when the crown was much reduced from its typical size for a given stem diameter because of crown loss (pruning, storm damage, senescence, etc.). This implies that actual load should play a role in assessing a given $t/R$ ratio, although nothing is said about the height at which that load is applied, a critique made very early in the development of the theory (Sinn and Wessolly 1989).

The Mattheck/Breloer formulation had two big advantages over the others:

- The mathematical level required of the practitioner was low: no raising numbers to the third power, no calculating the contents of parentheses, and no dividing big numbers. The level could be dropped even farther for field use with a simple approximation that everyone could understand and use: for every 6 inches (15 centimeters) of diameter, a tree needs about 1 inch (2.5 centimeters) of radial sound wood to avoid stem failure.
- The $t/R$ formula appeared to be supported by a large data set of “more than 1,200 broken and standing broadleaf and coniferous trees,” which, as represented by the well-known graph (Figure 1) through which it was published, seemed to indicate a clear separation between broken and standing trunks at the 0.3 threshold (though in the text, Mattheck and Breloer frequently mention a broader threshold range): As a result of ease of use and apparent field data support, the application of the $t/R$ requirement quickly became widespread among U.S. arborists carrying out risk assessment.
Current Status

An important reworking of the data set behind the $t/R$ graph has been published (Mattheck et al. 2006). The updated results come from the field studies (Mattheck et al. 1993) that formed the original portion of the data behind the 1994 publication. The significance of the reworking becomes visible when the data for broken stems are separated out from those for standing stems (Figure 2).

This updated graph differs greatly from Figure 1. We now see a distribution of $t/R$ values—that is to say, $t/R$ is revealed to be a discrete random variable (“random” in the statistical sense of mapping random experiments to numbers). In other words, no single $t/R$ value predicts stem failure. Like all random variables, it exists as a probability distribution, without any single, clear, catastrophic limit. The distribution illustrated in Figure 2 for this population indicates that one-third of hollow stems failed by the time the $t/R$ ratio reached 0.25—and, of course, that means two-thirds did not!

The proportion reverses for the next ratio class so that about two-thirds of hollow stems (based on those examined here) failed before the $t/R$ ratio reached 0.20. This number arises for analogous biological structures in general:

It should be noted that this mean $t/R$ ratio of 0.25–0.2 for broken stems differs significantly from the well-known conclusions of Smiley and Fraedrich (1992), who reported that 50% of the broken trees after a hurricane had a strength loss level greater than 33% (a $t/R$ ratio of about 0.3), though the actual weighted average of the data shown is about 37% (a $t/R$ ratio of about 0.27). The two studies are not strictly comparable. In contrast to Mattheck, Smiley and Fraedrich included trees with open cavities, examined a single storm event and a single genus, and dealt with a much smaller though much more carefully described population (Niklas 1992).

Support for the understanding that the $t/R$ ratio of failed stems is a random variable has come from another significant data set that recently became available to the English-speaking audience. The group Sachverständigenarbeitsgemeinschaft (SAG) für Baumstatik (Unified Consultants for Tree Statics) has been examining hollow and standing trees for the past two decades, following the research of Lothar Wessolly and others. In reviewing that group’s previously published data, Ditter et al. (2005) reproduced a graph of $t/R$ ratio by radius that summarizes close to 5,000 individual tree investigations (Figure 3).

A full 45 percent of this large number of trees “in parks and along roadsides” had a $t/R$ ratio less than 0.3. The trees not only were standing but also were stable, at least according to the pulling test protocol employed by the SAG group (Brudi and van Wassenhaer 2002).

We need to be careful about the conclusions we draw from these recent publications. Neither study was published in the standard Brazier, or “hose pipe,” buckling of a white ash ($Fraxinus americana$) in a forest. The tree had a $t/R$ ratio of about 0.1.

**Figure 1.** Trees plotted by $t/R$ against radius (Mattheck and Breloer 1994).

**Figure 2.** Distribution of the frequency of $t/R$ classes for 802 broken trees (recreated from Figure 1 in Mattheck et al. 2006, with x-axis converted).
Problems with the \( t/R \) Requirement

**Large Trees**
Practitioners often ignore the fact that the graph displays no support for applying the formula to trees with a diameter greater than about 36 inches (90 centimeters). Beyond that size limit, the proportion of standing larger stems to broken ones is, at best, 1:1, suggesting that, for larger trees, \( t/R \) is not a good predictor of whole stem failure.

**Other Tree Parts**
Other tree parts, such as codominant stems or buttress roots, are sometimes subjected to \( t/R \) analysis. Few would doubt that it is prudent to set an upper threshold for the risk assessment for such tree parts. But the engineering formula used in the evaluation procedures was derived from the resistance to stress in a hollow column and therefore may not apply to tree parts lacking a mechanically analogous structure.

**Precision**
The \( t/R \) graph appears to be amazingly precise: No failures occur above a \( t/R \) ratio of about 0.3, and, below that threshold, failure appears likely (judging from the dominance of black squares). Yet because the actual data were never published, standard statistical analysis for interpreting the significance of field data has not been possible, and we have little idea of what actually happens below the limit of 0.3.

**Unique Factor**
The user of \( t/R \) can only accept on faith the conclusion that the \( t/R \) ratio is sufficient to predict whole stem failure. Because the graph excludes other, possibly confounding factors (such as height, wind exposure, or species), the actual correlation between the two factors cannot be evaluated scientifically.

**Species**
The irrelevance of species in the \( t/R \) requirement appears to conflict with the practical experience that Wagener cited and that climbers still rely on. This conflict may arise because species differ in more ways than just wood strength.

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**Practical Implications**
- The ratio \( t/R \) can no longer be used by itself as an index of trunk failure potential.
- Trees can tolerate extremely large amounts of internal decay without necessarily incurring adverse effects on their stability.
- Given similar wind load, the tree with the lower \( t/R \) ratio will usually fail first, though actual wood properties may play a greater role here than often recognized by either Mattheck or Wessolly (Kane et al. 2001). The converse is also true: given similar \( t/R \) ratios, the tree with the greater actual load is more likely to fail.
- Trees with decurrent architecture are less likely to incur stem failure than those with excurrent architecture at the same \( t/R \) ratio.
ratio (Kane et al. 2001), apparently because of the strong mass damping (reduction of energy) carried out by large complex lateral branches (James 2003).

- The practitioner in front of a tree with a centered cavity (Kane and Ryan 2004) should use $t/R$ in conjunction with evaluation of other factors that contribute to failure: wind load, exposure, crown architecture, and species.

- Because few practitioners are trained in actual wind load analysis, there is a pressing need for a standard field method for the estimation of the hazard potential of decayed stems that does not require complex mathematics.

References


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