i-Tree Eco Dry Deposition Model Descriptions

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1. Introduction

Employing field-surveyed urban forest information, location specific data, weather data, and air pollutant measurements, i-Tree Eco assesses the structure of community trees and quantifies the environmental services that trees provide. i-Tree Eco was developed based on the Urban Forest Effects (UFORE) model. With its dry deposition component (UFORE-D) integrated into i-Tree Eco, dry deposition of air pollution (i.e., pollution removal during nonprecipitation periods) to trees and shrubs and associated percent improvement in air quality can be estimated with i-Tree Eco. The dry deposition of criteria air pollutants (CAPs); carbon monoxide (CO), nitrogen dioxide (NO\textsubscript{2}), ozone (O\textsubscript{3}), sulfur dioxide (SO\textsubscript{2}), and particulate matter less than 10 microns (PM10) can be assessed with version 4. In addition, particulate matter less than 2.5 microns (PM2.5) can be assessed with version 5. This document provides detailed i-Tree Eco dry deposition model descriptions.

2. Model Descriptions

i-Tree Eco provides functions to calculate values listed below. These functions are described in this section.

- Air pollutant flux calculation
- Deposition velocity calculation
- Aerodynamic resistance calculation
- Friction velocity calculation
- Quasi-laminar boundary layer resistance calculation
- Canopy resistance calculation
- Stomatal resistance calculation
- Transpiration calculation
- Air quality improvement calculation
- Air pollutant concentration change calculation
- Monetary value calculation

2.1. Air pollutant flux calculation for CO, NO\textsubscript{2}, SO\textsubscript{2}, O\textsubscript{3} and PM10

Pollutant flux is calculated as a product of the deposition velocity and the air pollutant
concentration:

\[ F = V_d \cdot C \cdot 3600 \]  \hspace{1cm} (1)
\[ F_{min} = V_{d,min} \cdot C \cdot 3600 \]  \hspace{1cm} (2)
\[ F_{max} = V_{d,max} \cdot C \cdot 3600 \]  \hspace{1cm} (3)

where

\[ F \] = Pollutant flux (g m\(^{-2}\) h\(^{-1}\))
\[ F_{min} \] = Minimum pollutant flux (g m\(^{-2}\) h\(^{-1}\))
\[ F_{max} \] = Maximum pollutant flux (g m\(^{-2}\) h\(^{-1}\))
\[ V_d \] = Deposition velocity (m s\(^{-1}\))
\[ V_{d,min} \] = Minimum deposition velocity (m s\(^{-1}\))
\[ V_{d,max} \] = Maximum deposition velocity (m s\(^{-1}\))
\[ C \] = Air pollutant concentration (g m\(^{-3}\))

2.2. Deposition velocity calculation for CO, NO\(_2\), SO\(_2\), and O\(_3\)

Deposition velocities (\(V_d\)) for CO, NO\(_2\), SO\(_2\), and O\(_3\) are calculated as the inverse of the sum of the aerodynamic resistance (\(R_a\)), quasi-laminar boundary layer resistance (\(R_b\)), and canopy resistance (\(R_c\)) (Baldocchi et al. 1987). The aerodynamic resistance can be calculated with meteorological data and thus is independent of the air pollutant types, while the quasi-laminar boundary layer and canopy resistances are separately calculated for CO, NO\(_2\), O\(_3\), and SO\(_2\). In addition, the canopy resistance is calculated depending on in-leaf and out-of-leaf periods.

\[ V_d = \frac{1}{R_a + R_b + R_c} \]  \hspace{1cm} (4)

where

\[ V_d \] = Deposition velocity (m s\(^{-1}\))
\[ R_a \] = Aerodynamic resistance (s m\(^{-1}\))
\[ R_b \] = Quasi-laminar boundary layer resistance for a type of air pollutions (s m\(^{-1}\))
\[ R_c \] = Canopy resistance (s m\(^{-1}\))
Depending on the flux density of photosynthetically active radiation (PAR), minimum and maximum of $V_d$ are estimated as follows (Lovett 1994):

If PAR > 0 (in the daytime)

for NO$_2$

$$V_{d,min} = 0.001$$

$$V_{d,min} = 0.005$$

(5)

for O$_3$

$$V_{d,min} = 0.001$$

$$V_{d,min} = 0.008$$

(6)

for SO$_2$

$$V_{d,min} = 0.002$$

$$V_{d,min} = 0.01$$

(7)

else (in the nighttime)

$$V_{d,min} = V_d$$

$$V_{d,min} = V_d$$

(8)

2.3. Deposition velocity calculation for PM10

Deposition velocity for PM10 is calculated based on average, minimum, and maximum values reported by Lovett (1994).

$$V_d = V_{d,\text{PM10,avg}} \cdot \frac{\text{BAI+LAI}}{\text{BAI+LAI}_{\text{PM10}}}$$

(9)

$$V_{d,\text{min}} = V_{d,\text{PM10,min}} \cdot \frac{\text{BAI+LAI}}{\text{BAI+LAI}_{\text{PM10}}}$$

(10)

$$V_{d,\text{max}} = V_{d,\text{PM10,max}} \cdot \frac{\text{BAI+LAI}}{\text{BAI+LAI}_{\text{PM10}}}$$

(11)
where

\[ V_{d,PM10,avg} = \text{Average deposition velocity for PM10 (}= 0.0064 \text{ ms}^{-1}) \] (Lovett 1994)
\[ V_{d,PM10,min} = \text{Minimum deposition velocity for PM10 (}= 0.0025 \text{ ms}^{-1}) \] (Lovett 1994)
\[ V_{d,PM10,max} = \text{Maximum deposition velocity for PM10 (}= 0.01 \text{ ms}^{-1}) \] (Lovett 1994)
\[ LAI_{PM10} = \text{Leaf area index for particle deposition (}= 6) \]
\[ BAI = \text{Bark area index} \]
\[ LAI = \text{Leaf area index} \]

### 2.4. Deposition velocity calculation for PM2.5

Deposition velocities of PM2.5 to trees were estimated from literature and varied with wind speed (Beckett et al. 2000, Freer-Smith et al. 2004, Pullman 2009). These papers measured deposition velocities to tree leaves from 17 tree species under wind speeds of 1, 3, 6, 8, 9 and 10 m s\(^{-1}\). For each wind speed, the median deposition velocities from the measured deposition velocities was used to estimate the \( V_{d,PM2.5} \) for that wind speed per unit leaf area (Table 1).

\[ V_d = V_{d,PM2.5} \cdot LAI \quad (12) \]

The standard error of the estimates among the species was used to estimate a potential range of values of deposition velocities. The median deposition velocity per wind speed plus 1.96 times the standard error was used to estimate a maximum deposition for the wind speed (\( V_{d,PM2.5,Max} \)). As using standard errors to estimate the lower range of deposition velocities produced negative deposition velocities, the minimum average \( V_d \) from any species was used to represent the minimum \( V_d \) for the wind speed (\( V_{d,PM2.5,Min} \)). To estimate the \( V_d \) for wind speeds between 1 and 10 m s\(^{-1}\) that did not have a measured \( V_d \), values were interpolated between the closest measured values. For wind speeds above 10 m s\(^{-1}\), the \( V_d \) for 10 m s\(^{-1}\) was used; for a wind speed of 0 m s\(^{-1}\), the \( V_d \) was assumed to be 0 cm s\(^{-1}\) (Table 2).

\[ V_{d,min} = V_{d,PM2.5,min} \cdot LAI \quad (13) \]
\[ V_{d,max} = V_{d,PM2.5,max} \cdot LAI \quad (14) \]

### Table 1 Summary of average deposition velocities (cm/sec) of PM2.5 by wind speed from the literature per unit leaf area

<table>
<thead>
<tr>
<th>Species</th>
<th>Wind Speed (m s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Species</td>
<td>1</td>
</tr>
<tr>
<td>---------------------------------</td>
<td>-------</td>
</tr>
<tr>
<td><em>Quercus petraea</em>&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.831</td>
</tr>
<tr>
<td><em>Alnus glutinosa</em>&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.125</td>
</tr>
<tr>
<td><em>Fraxinus excelsior</em>&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.178</td>
</tr>
<tr>
<td><em>Acer pseudoplatanus</em>&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.042</td>
</tr>
<tr>
<td><em>Psuedotsuga menziesii</em>&lt;sup&gt;b&lt;/sup&gt;</td>
<td>1.269</td>
</tr>
<tr>
<td><em>Eucalyptus globulus</em>&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.018</td>
</tr>
<tr>
<td><em>Ficus nitida</em>&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.041</td>
</tr>
<tr>
<td><em>Pinus nigra</em>&lt;sup&gt;c&lt;/sup&gt;</td>
<td>0.13</td>
</tr>
<tr>
<td><em>Cupressocyparis x leylandii</em>&lt;sup&gt;c&lt;/sup&gt;</td>
<td>0.08</td>
</tr>
<tr>
<td><em>Acer campestre</em>&lt;sup&gt;c&lt;/sup&gt;</td>
<td>0.03</td>
</tr>
<tr>
<td><em>Sorbus intermedia</em>&lt;sup&gt;c&lt;/sup&gt;</td>
<td>0.04</td>
</tr>
<tr>
<td><em>Populus deltoides</em>&lt;sup&gt;c&lt;/sup&gt;</td>
<td>0.03</td>
</tr>
<tr>
<td><em>Pinus strobus</em>&lt;sup&gt;d&lt;/sup&gt;</td>
<td>0.0108</td>
</tr>
<tr>
<td><em>Tsuga canadensis</em>&lt;sup&gt;d&lt;/sup&gt;</td>
<td>0.0193</td>
</tr>
<tr>
<td><em>Tsuga japonica</em>&lt;sup&gt;d&lt;/sup&gt;</td>
<td>0.0058</td>
</tr>
<tr>
<td><em>Picea abies</em>&lt;sup&gt;e&lt;/sup&gt;</td>
<td>0.0189</td>
</tr>
<tr>
<td><em>Picea abies</em>&lt;sup&gt;e&lt;/sup&gt;</td>
<td>0.038</td>
</tr>
<tr>
<td>Median</td>
<td>0.030</td>
</tr>
<tr>
<td>SE&lt;sup&gt;f&lt;/sup&gt;</td>
<td>0.012</td>
</tr>
<tr>
<td>Maximum&lt;sup&gt;g&lt;/sup&gt;</td>
<td>0.057</td>
</tr>
<tr>
<td>Minimum&lt;sup&gt;h&lt;/sup&gt;</td>
<td>0.006</td>
</tr>
</tbody>
</table>

<sup>a</sup> combination of 8 and 9 m s<sup>-1</sup> wind speeds
<sup>b</sup> from Freer-smith (2004)
<sup>c</sup> from Beckett et al (2000)
<sup>d</sup> from Pullman (2009). Included particles up to 3.0 µm in diameter
<sup>e</sup> from Pullman (2009). Based on maximum and minimum of reported range. Included particles up to 3.8 µm in diameter.
<sup>f</sup> standard error
<sup>g</sup> based on median plus one standard error
<sup>h</sup> based on lowest recorded value for any species

Resuspension of PM2.5 from trees were estimated from Pullman (2009) and varied with wind speed. This paper measured percent resuspension of PM<sub>2.5</sub> from tree leaves of three tree species.
under wind speeds of 6.5, 10 and 13 m s\(^{-1}\). The average percent resuspension for the trees species and wind speed was calculated (Table 2). As the percent resuspension for the wind speed of 6.5 m s\(^{-1}\) was 9.5%, a value of 9% was assumed for a wind speed of 6 m s\(^{-1}\) and 10% for 7 m s\(^{-1}\). The percent resuspension for a wind speed of 0 m s\(^{-1}\) was assumed to be 0%. To estimate the percent resuspension for wind speeds between 0 and 13 m s\(^{-1}\) that did not have a measured resuspension rates, values were interpolated between the closest measured values (or assumed value at wind speed of 0 m s\(^{-1}\)). For wind speeds above 13 m s\(^{-1}\), the percent resuspension rate for 13 m s\(^{-1}\) was used (Table 2).

<table>
<thead>
<tr>
<th>Wind speed (m s(^{-1}))</th>
<th>Deposition Velocity (V_d)</th>
<th>Resuspension (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average (cm s(^{-1}))</td>
<td>Minimum (cm s(^{-1}))</td>
</tr>
<tr>
<td>0</td>
<td>0.00</td>
<td>0.000</td>
</tr>
<tr>
<td>1</td>
<td>0.03</td>
<td>0.006</td>
</tr>
<tr>
<td>2</td>
<td>0.09</td>
<td>0.012</td>
</tr>
<tr>
<td>3</td>
<td>0.15</td>
<td>0.018</td>
</tr>
<tr>
<td>4</td>
<td>0.17</td>
<td>0.022</td>
</tr>
<tr>
<td>5</td>
<td>0.19</td>
<td>0.025</td>
</tr>
<tr>
<td>6</td>
<td>0.20</td>
<td>0.029</td>
</tr>
<tr>
<td>7</td>
<td>0.56</td>
<td>0.056</td>
</tr>
<tr>
<td>8</td>
<td>0.92</td>
<td>0.082</td>
</tr>
<tr>
<td>9</td>
<td>0.92</td>
<td>0.082</td>
</tr>
<tr>
<td>10</td>
<td>2.11</td>
<td>0.570</td>
</tr>
<tr>
<td>11</td>
<td>2.11</td>
<td>0.570</td>
</tr>
<tr>
<td>12</td>
<td>2.11</td>
<td>0.570</td>
</tr>
<tr>
<td>13</td>
<td>2.11</td>
<td>0.570</td>
</tr>
</tbody>
</table>

2.5. Air pollutant flux calculation for PM2.5

During non-precipitation periods, flux values of PM2.5 are cumulated on leaves hourly with a percent of the accumulated PM2.5 resuspended back to the atmosphere based on local wind.
speed. At time $t$, newly deposited PM2.5 flux can be calculated as

$$f_t = V_{d,PM2.5,t} \cdot C_t \cdot 3600$$  \hspace{1cm} (15)

$$f_{min,t} = V_{d,PM2.5,\text{min},t} \cdot C_t \cdot 3600$$  \hspace{1cm} (16)

$$f_{max,t} = V_{d,PM2.5,\text{max},t} \cdot C_t \cdot 3600$$  \hspace{1cm} (17)

where

- $f_t = \text{PM2.5 flux at time } t \text{ (g m}^{-2}\text{ h}^{-1})$
- $f_{min} = \text{Minimum PM2.5 flux at time } t \text{ (g m}^{-2}\text{ h}^{-1})$
- $f_{max} = \text{Maximum PM2.5 flux at time } t \text{ (g m}^{-2}\text{ h}^{-1})$
- $V_{d,PM2.5,t} = \text{Deposition velocity at time } t \text{ (m s}^{-1})$
- $V_{d,PM2.5,\text{min},t} = \text{Minimum deposition velocity at time } t \text{ (m s}^{-1})$
- $V_{d,PM2.5,\text{max},t} = \text{Maximum deposition velocity at time } t \text{ (m s}^{-1})$
- $C_{adj,t} = \text{Air pollutant concentration at time } t \text{ (g m}^{-3})$

Resuspended amount of PM2.5 at time $t$ can be calculated as

$$R_t = (A_{t-1} + f_t) \cdot \frac{rr_t}{100}$$  \hspace{1cm} (18)

$$R_{min,t} = (A_{\text{min},t-1} + f_{min,t}) \cdot \frac{rr_t}{100}$$  \hspace{1cm} (19)

$$R_{max,t} = (A_{\text{max},t-1} + f_{max,t}) \cdot \frac{rr_t}{100}$$  \hspace{1cm} (20)

where

- $R_t = \text{PM2.5 flux resuspended to atmosphere at time } t \text{ (g m}^{-2}\text{ h}^{-1})$
- $R_{min} = \text{Minimum PM2.5 flux resuspended to atmosphere at time } t \text{ (g m}^{-2}\text{ h}^{-1})$
- $R_{max} = \text{Maximum PM2.5 flux resuspended to atmosphere at time } t \text{ (g m}^{-2}\text{ h}^{-1})$
- $A_{t-1} = \text{PM2.5 accumulated on leaves at time } t \text{ (g m}^{-2}\text{ h}^{-1})$
- $A_{\text{min},t-1} = \text{Minimum PM2.5 accumulated on leaves at time } t \text{ (g m}^{-2}\text{ h}^{-1})$
- $A_{\text{max},t-1} = \text{Maximum PM2.5 accumulated on leaves at time } t \text{ (g m}^{-2}\text{ h}^{-1})$
- $rr_t = \text{APM2.5 resuspension rate at time } t \text{ (%)$}
After PM2.5 resuspended back to the atmosphere, accumulated amount on leaves at time $t$ can be calculated as

$$A_t = (A_{t-1} + f_t) - R_t$$  \hspace{1cm} (21)

$$A_{\text{min},t} = (A_{\text{min},t-1} + f_{\text{min},t}) - R_{\text{min},t}$$ \hspace{1cm} (22)

$$A_{\text{max},t} = (A_{\text{max},t-1} + f_{\text{max},t}) - R_{\text{max},t}$$ \hspace{1cm} (23)

After PM2.5 resuspended back to the atmosphere, net PM2.5 flux at time $t$ can be calculated as

$$F_t = f_t - R_t$$ \hspace{1cm} (24)

$$F_{\text{min},t} = f_{\text{min},t} - R_{\text{min},t}$$ \hspace{1cm} (25)

$$F_{\text{max},t} = f_{\text{max},t} - R_{\text{max},t}$$ \hspace{1cm} (26)

where

$F_t =$ Net PM2.5 flux at time $t$ (g m$^{-2}$ h$^{-1}$)

$F_{\text{min}} =$ Minimum net PM2.5 flux at time $t$ (g m$^{-2}$ h$^{-1}$)

$F_{\text{max}} =$ Maximum net PM2.5 flux at time $t$ (g m$^{-2}$ h$^{-1}$)

When the net flux is negative due to a large amount of resuspension and the absolute value of the net flux is greater than the total PM2.5 amount in the atmosphere ($M_{\text{total}}$ (g m$^{-2}$ h$^{-1}$) = $C$ (g m$^{-3}$ h$^{-1}$) × mixing layer height (m)), the net flux is adjusted to be equal to -$M$ (g m$^{-2}$ h$^{-1}$).

During precipitation events, the accumulated PM2.5 is assumed to be washed off to the ground surface, depending upon the precipitation intercepted and stored on leaves and running off from leaves. Until it reaches its maximum capacity of storage (0.2mm × LAI) (Wang et al. 2008, Hirabayashi 2012), precipitation is stored on leaves and no water runoffs from leaves. During this period, no PM2.5 is washed off. Once it reaches the maximum, water on leaves starts running off, with which all of the accumulated PM2.5 are assumed to be washed off. Thus, resuspension, accumulation and net flux for these time periods become 0.
2.6. Aerodynamic resistance calculation

The aerodynamic resistance \( R_a \) is calculated as (Killus et al. 1984):

\[
R_a = \frac{u(z)}{u^2}
\]  

(27)

where

- \( u(z) \) = Mean wind speed at height \( z \) (ms\(^{-1}\))
- \( u^* \) = Friction velocity (ms\(^{-1}\))

2.7. Friction velocity calculation

Depending on the stability of atmosphere (i.e., neutral, unstable, and stable), \( u^* \) is calculated as described in the following sections. The stability of atmosphere can be determined by Monin-Obuhkov stability length, which can be empirically estimated based on the Pasquill stability class.

\[
\text{Pasquill} = A: \frac{1}{L} = -0.0875 \cdot z_o^{-0.1029}
\]

\[
\text{Pasquill} = B: \frac{1}{L} = -0.03849 \cdot z_o^{-0.1714}
\]

\[
\text{Pasquill} = C: \frac{1}{L} = -0.0807 \cdot z_o^{-0.3049}
\]

\[
\text{Pasquill} = D: \frac{1}{L} = 0 \cdot z_o^0
\]

\[
\text{Pasquill} = E: \frac{1}{L} = 0.0807 \cdot z_o^{-0.3049}
\]

\[
\text{Pasquill} = F: \frac{1}{L} = 0.03849 \cdot z_o^{-0.1714}
\]

(28)

where

- \( L \) = Monin-Obuhkov stability length
- \( z_o \) = Roughness length
2.7.1. Neutral atmosphere (L=0)

For the neutral atmosphere, $u_*$ is calculated as:

$$u_* = \frac{k \cdot u(z-d)}{\ln \left( \frac{z-d}{z_0} \right)} \quad (29)$$

where

- $k$ = von Karman constant (=0.41)
- $u(z)$ = Mean wind speed at height $z$ (m s$^{-1}$)
- $z$ = Height of the weather station (m)
- $d$ = Displacement height (m)
- $z_o$ = Roughness length (m)

2.7.2. Unstable atmosphere (L < 0)

When the atmosphere is unstable such as during daytime when the air convection occurs, $u_*$ is calculated as (Killus et al. 1984):

$$u_* = \frac{k \cdot u(z-d)}{\ln \left( \frac{z-d}{z_0} \right) - \psi_M \left( \frac{z-d}{L} \right) + \psi_M \left( \frac{z_0}{L} \right)} \quad (30)$$

where

- $\psi_M$ : Stability function for momentum
- $L$ : Monin-Obukhov stability length

Stability function for momentum ($\psi_M$) is calculated as (van Ulden and Holtslag 1985):

$$\psi_M = 2\ln \left( \frac{1+x}{2} \right) + \ln \left( \frac{1+x^2}{2} \right) - 2\tan^{-1}(x) + \frac{\pi}{2} \quad (31)$$

where $x$ is calculated as (Dyer and Bradley 1982):

$$x = \frac{1}{\left( 1 - 2\theta \frac{z}{L} \right)^{0.25}} \quad (32)$$

2.7.3. Stable atmosphere (L > 0)

When the atmosphere is stable $u_*$ is calculated as (Venkatram 1980):
\[ u_* = C_{DN} \cdot u(z) \left[ \frac{1}{2} + \frac{1}{2} \sqrt{1 - \left( \frac{2u_o}{\sqrt{C_{DN}\cdot u(z)}} \right)^2} \right] \]  

(33)

where

\( C_{DN} \) = Neutral drag coefficient (dimensionless)

\( C_{DN} \) is calculated as (US EPA 1995):

\[ C_{DN} = \frac{k}{\ln \left( \frac{z}{z_o} \right)} \]  

(34)

\( u_o \) is calculated as (US EPA 1995):

\[ u_o = \sqrt{\frac{\beta_m z g \theta_*}{T}} \]  

(35)

where

\( \beta_m \) = Dimensionless constant (= 4.7)

\( g \) = Acceleration due to gravity (= 9.81 ms\(^{-2}\))

\( T \) = Air temperature (K)

\( \theta_* \) is calculated as (US EPA 1995)

\[ \theta_* = 0.09(1 - 0.5N^2) \]  

(36)

where

\( N \) = Fraction of opaque cloud cover

To obtain real-valued solutions for \( u_* \), the following condition must hold

\[ \frac{2u_o}{\sqrt{C_{DN}\cdot u(z)}} \leq 1 \]  

(37)

Otherwise, the friction velocity (\( u_* \)) is calculated as (US EPA 1995):

\[ u_* = u_{*cr} \cdot \frac{u(z)}{u_{cr}} \]  

(38)

where \( u_{cr} \) and \( u_{*cr} \) are calculated as (US EPA 1995):
\[ u_{cr} = \frac{4}{\sqrt{C_{DN}}} u_o \]  
\[ u_{*cr} = \frac{C_{DN} u_{cr}}{2} \]  

2.8. Quasi-laminar boundary layer resistance calculation

Quasi-laminar boundary layer resistance \( R_b \) is calculated as (Pederson et al. 1995)

\[ R_b = 2 (Sc)^{2/3} (Pr)^{2/3} (k u_*)^{-1} \]  

where

\[ Sc = \text{Schmidt number} \]
\[ Pr = \text{Prandtl number} (= 0.72) \]

The Schmidt number is equal to 1 for \( O_3 \), 0.76 for CO, 0.98 for \( NO_2 \), and 1.15 for \( SO_2 \).

2.9. Canopy resistance calculation for CO

2.9.1. In-leaf periods

As removal of CO by vegetation is not directly related to transpiration, the canopy resistance for CO in in-leaf periods \( (R_{c,\text{InLeaf}}) \) is set to a constant based on data from Bidwell and Fraser (1972).

\[ R_{c,\text{InLeaf}} = 50,000 \quad (\text{sm}^{-1}) \]  

2.9.2. Out-of-leaf Periods

The canopy resistance for CO in out-of-leaf periods \( (R_{c,\text{OutLeaf}}) \) is set to a constant based on data from Bidwell and Fraser (1972).

\[ R_{c,\text{OutLeaf}} = 1,000,000 \quad (\text{sm}^{-1}) \]  

2.10. Canopy resistance calculation for \( NO_2 \), \( O_3 \), and \( SO_2 \)

The canopy resistances for \( NO_2 \), \( O_3 \), and \( SO_2 \) \( (R_c) \) can be calculated as
\[ \frac{1}{R_c} = \frac{1}{r_s + r_m} + \frac{1}{r_{soil}} + \frac{1}{r_t} \]  

(44)

where

- \( r_s \) = Stomatal resistance
- \( r_m \) = Mesophyll resistance
- \( r_{soil} \) = Soil resistance (= 2941 s m\(^{-1}\) in growing season, =2000 s m\(^{-1}\) otherwise)
- \( r_t \) = Cuticular resistance

\( r_m \) and \( r_t \) for NO\(_2\), O\(_3\), and SO\(_2\) are presented in Table 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>NO(_2)</th>
<th>O(_3)</th>
<th>SO(_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_m )</td>
<td>100 (s m(^{-1}))</td>
<td>10 (s m(^{-1}))</td>
<td>0 (s m(^{-1}))</td>
</tr>
<tr>
<td>( r_t )</td>
<td>20,000 (s m(^{-1}))</td>
<td>10,000 (s m(^{-1}))</td>
<td>8,000 (s m(^{-1}))</td>
</tr>
</tbody>
</table>

### 2.11. Stomatal resistance calculation

The visible part (400 – 700 nm) or photosynthetically active radiation (PAR) of the solar spectrum is largely absorbed to drive photosynthesis, while the near-infrared part (700 – 3000 nm) is largely scattered by vegetations (Norman 1982). Direct beam and sky diffuse radiation are intercepted very differently by canopies (Norman 1982). Diffuse radiation is used much more efficiently than direct radiation by a canopy of photosynthesizing leaves (Weiss and Norman 1985). The stomatal resistance is linked to leaf photosynthesis (Baldocchi 1994). Calculating canopy photosynthetic rates from leaf photosynthetic rates requires dividing the canopy into N layers, in which \( \Delta F \), the leaf area index in each layer, usually chosen as 0.1 or 0.2 (Norman 1980). It is preferable to compute stomatal resistance on the basis of the irradiance on both sunlit and shaded leaves and weighting these resistances according to the fraction of sunlit and shaded leaf area (Baldocchi et al., 1987). Therefore, derivation of stomatal conductance \( g_s \), the inverse of stomatal resistance of a
canopy involves these steps, which are explained in the following sections.

1. Divide solar radiation above the canopy into visible (PAR) and near-infrared portions (Section 2.11.1).
2. Divide PAR above the canopy into direct beam and diffuse radiation (Section 2.11.1).
3. Divide the canopy into N layers and calculate sunlit and shaded leaf areas in each layer of the canopy (Section 2.11.2).
4. Calculate flux density of PAR intercepted by sunlit and shaded leaves in each layer of the canopy (Section 2.11.3).
5. Calculate stomatal conductance for sunlit and shaded leaves in each layer of the canopy (Sections 2.11.4 and 2.11.5).
6. Weight stomatal conductance for each layer according to the fraction of sunlit and shaded leaf area (Section 2.11.6).
7. Accumulate stomatal conductance for sunlit and shaded leaves throughout all layers to derive stomatal conductance for the whole canopy (Section 2.11.7).

2.11.1. Visible and near-infrared, direct and diffuse components of solar incident

Weiss and Norman (1985) obtained equations to predict direct and diffuse components of solar irradiance in both the visible (PAR) and near-infrared wave bands from measurements of only the total incoming solar radiation. Direct visible component ($R_{DV}$), diffuse visible component ($R_{dV}$), direct near-infrared component ($R_{DN}$), and diffuse near-infrared component ($R_{dN}$) of the solar radiation can be approximated by (Weiss and Norman 1985):

\[
R_{DV} = 600 \left( -0.185 \cdot \frac{P}{P_0} \cdot m \right) \cos \theta 
\]

\[
R_{dV} = 0.4(600 - R_{DV}) \cos \theta 
\]

\[
R_{DN} = \left[ 720 \exp \left( -0.06 \frac{P}{P_0} m \right) - w \right] \cos \theta 
\]

\[
R_{dN} = 0.6(720 - R_{DN} - w) \cos \theta 
\]

where

\[
E = \text{Extinction coefficient (}= 0.185) \\
P = \text{Actual pressure (kPa)} \\
P_0 = \text{Sea level pressure (= 101.325 kPa)}
\]
\( m = \) Optical air mass

With a flat approximation to the atmosphere, \( m \) can be defined as:

\[
m = \frac{1}{\cos \theta}
\]  

(49)

\( \theta = \) Solar zenith angle
\( w = \) Water absorption in the near infrared for 10 mm of precipitable water

\( w \) can be derived from

\[
w = 1320 \text{antilog}_{10}[-1.1950 + 0.4459 \log_{10} m - 0.0345 (\log_{10} m)^2]
\]  

(50)

The fraction of the visible (or near-infrared) in the direct beam was estimated by analyzing the relationship between the fraction and the ratio of measured to potential total solar radiation, \( RATIO \) (Weiss and Norman 1985).

\[
f_V = \frac{R_{DV}}{R_V} \left[ 1 - \left( \frac{A - RATIO}{B} \right)^{\frac{2}{3}} \right]
\]  

(51)

\[
f_N = \frac{R_{DN}}{R_N} \left[ 1 - \left( \frac{C - RATIO}{D} \right)^{\frac{2}{3}} \right]
\]  

(52)

\[
RATIO = \frac{R_T}{R_V + R_N}
\]  

(53)

where
\( R_T = \) Measurement of total incoming solar radiation
\( R_V = R_{DV} + R_{dV} \)
\( R_N = R_{DN} + R_{dN} \)

\( A, B, C, \) and \( D \) are 0.9, 0.7, 0.88, and 0.68, respectively. \( RATIO \) is never allowed to exceed \( A \) or \( C \). If \( RATIO \) should exceed \( A \) or \( C \), it is set equal to \( A \) or \( C \).

It is assumed that 46% of the averaged hourly solar radiation is in the visible (Norman 1982). Norman (1982) states this percentage is essentially independent of zenith angle and cloudiness. Flux densities of direct and diffuse radiation of PAR on top of the canopy can be estimated as
\[ \text{PAR}_{\text{dir}} = f_V(0.46R_T)F_c \]  
\[ \text{PAR}_{\text{diff}} = (1 - f_V)(0.46R_T)F_c \]  

where

- \( F_c \) = Conversion factor from W m\(^{-2}\) to \( \mu \text{E} \) m\(^{-2}\) s\(^{-1}\) (=4.6)

\( F_c \) was derived by converting energy of solar irradiance into photosynthetic photon flux density (PPFD), which is a measure of the number of photons in the visible spectrum range falling on a 1 m\(^2\) area per second. PPFD is a measure of PAR. The energy carried by electromagnetic radiation is contained in the photons that travel as a wave. The energy in a photon varies with its frequency, according to the equation:

\[ E = h\nu = \frac{hc}{\lambda} \]  

- \( E \) = Energy (J)
- \( h \) = Plank’s constant (= 6.626 \( \times \) 10\(^{-34}\) J s\(^{-1}\))
- \( \nu \) = Frequency
- \( c \) = Speed of light (= 3.0 \( \times \) 10\(^8\) m s\(^{-1}\))
- \( \lambda \) = Wavelength (m)

The conversion is conducted based on the center wavelength (=550 nm) of the visible part of the spectrum. As one photon in a certain wavelength has the energy calculated by Eq. 4-xx, the number of photons in 550 nm comprising one joule (J) can be calculated as 27.70\( \times \) 10\(^{17}\). This number can be converted into as 4.59\( \times \) 10\(^{-6}\) moles where one mole = Avogadro’s number = 6.02\( \times \) 10\(^{23}\). Watt (W) is a unit of power that measures rate of flow of energy. One (W) is equivalent to one (J) of energy per second. Thus, one (W) of power from light at 550 nm would need to provide 27.70\( \times \) 10\(^{17}\) photons, or 4.6\( \times \) 10\(^{-6}\) moles per second. Another unit, Einstein (E) is introduced here, which is used in irradiance and in photochemistry. One einstein is defined as one mole of photons, regardless of their frequency. One (W) of power from light would need to provide 4.6\( \times \) 10\(^{-6}\) (E) or 4.6 (\( \mu \text{E} \)) per second. Therefore, irradiance in W m\(^{-2}\) can be converted into PPFD in \( \mu \text{E} \) m\(^{-2}\) s\(^{-1}\) by multiplying 4.6.

### 2.11.2. Canopy layers and sunlit/shaded leaf area index for each layer

The canopy is divided into N layers, in which the leaf area index in each layer (\( \Delta F \)) chosen as 0.1 or 0.2, to calculate the PAR on sunlit and shaded leaves separately, assuming a mean
angle between the direction of the sun and the plane of sunlit leaves and the spherical leaf angle distribution. This procedure is based on methods explained in Norman (1980). The direct beam transmittance below layer \( j \) of the canopy with a spherical leaf distribution is

\[
T_{B,j} = \exp \left( -\frac{F_j}{2\cos\theta} \right)
\]

(57)

where

\[
F_j = \text{Leaf area index at the layer } j
\]

\[
\theta = \text{Zenith angle of the sun}
\]

For a spherical leaf distribution, the sunlit leaf area index in the \( j^{th} \) layer is given by \( \Delta F^*_j \)

\[
\Delta F^*_j = (T_{B,j} - T_{B,j+1})2\cos\theta
\]

(58)

The fraction of leaf area in the \( j^{th} \) layer that is sunlit is given by

\[
\frac{\Delta F^*_j}{\Delta F}
\]

(59)

the shaded leaf area index in the \( j^{th} \) layer is given by

\[
\Delta F - \Delta F^*_j
\]

(60)

2.11.3. Flux density of PAR on sunlit/shaded leaves in each layer

Flux density of PAR on sunlit and shaded leaves can be estimated with methods outlined in Norman (1982) and Baldocchi et al. (1987). Shaded leaves receive only diffuse light. The flux density of diffuse PAR on shaded leaves in the \( j^{th} \) layer can be estimated as

\[
PAR_{shade,j} = PAR_{diff} \exp(-0.5F^{0.7}) \cdot S_j + C_j
\]

(61)

\[
PAR_{diff} = \text{Flux density of diffuse PAR above the canopy}
\]

\[
F = \text{Leaf area index for whole canopy}
\]

\[
S_j = \text{Scaling factor of diffuse flux density of PAR for the } j^{th} \text{ layer}
\]

\[
S_j = \frac{\exp(-0.5F^{j-0.5})}{\sum_{j=1}^{N} \exp(-0.5F^{j-0.5})}
\]

(62)

C arises from multiple scattering of direct beam radiation and is given by
Conceptually, the 0.07 represents a scattering coefficient, the term in brackets accounts for the decrease in multiple scattering with depth and the last exponential term accounts for the increased scattering at high zenith angles.

Sunlit leaves receive both direct and diffuse lights. The flux density of PAR on sunlit leaves in the \( j \)th layer can be estimated as

\[
PAR_{\text{sun},j} = PAR_{\text{dir}} \frac{\cos \alpha \sin \beta}{\sin \beta} + PAR_{\text{shade},j}
\]  

where
- \( PAR_{\text{dir}} \) = Flux density of direct PAR above the canopy
- \( \alpha \) = Angle between a leaf and the sun (= 60 degrees)
- \( \beta \) = Solar elevation angle

\( PAR_{\text{dir}}/\cos \beta \) is the visible irradiance on a plane perpendicular to the direction of the sun.

2.11.4. Stomatal conductance for sunlit/shaded leaves in each layer

This procedure is based on methods explained in Farquhar et al. (1980), Baldoci (1994), and Harley et al. (1992). Stomatal conductance \( g_s \) for each layer of the canopy can be computed as:

\[
\frac{mArh}{C_s} + b'
\]

where
- \( m \) = Dimensionless slope (= 10)
- \( A \) = Photosynthetic flux density of a leaf
- \( rh \) = Relative humidity
- \( b' \) = Zero intercept when \( A \) is equal to or less than zero (= 0.02 mol m\(^{-2}\) s\(^{-1}\))
- \( C_s \) = Leaf surface CO\(_2\) concentration

\( A \) is represented as:

\[
A = V_c - 0.5V_o - R_d
\]

where
\[ V_c = \text{Carboxylation rate of CO}_2 \text{ exchange between leaf and atmosphere} \]
\[ V_o = \text{Oxygenation rate of CO}_2 \text{ exchange between leaf and atmosphere} \]
\[ R_d = \text{Dark respiration rate of CO}_2 \text{ exchange between leaf and atmosphere} \]

Dark respiration rate of CO\(_2\) exchange between leaf and atmosphere, \(R_d\), can be calculated as
\[
R_d = \frac{V_c(25^\circ C) \times 0.015 \times e^{\left[\frac{(T-298)E}{298RT}\right]}}{1+e^{\left[1.3(T-328)\right]}} \tag{67}
\]

\[ V_c(25^\circ C) = \text{Carboxylation rate of CO}_2 \text{ exchange between leaf and atmosphere (=90)} \]
\[ E = \text{Relevant activation energy} \]
\[ R = \text{Universal gas constant} \]
\[ T = \text{Absolute leaf temperature} \]

The term \(V_c - 0.5V_o\) is expressed by Baldocchi (1994) as
\[
V_c - 0.5V_o = \min(W_c, W_j) \left(1 - \frac{\Gamma}{C_i}\right) \tag{68}
\]

where
\[ W_c = \text{Carboxylation rate when ribulose bisphosphate (RuBP) carboxylase/oxygenase is saturated} \]
\[ W_j = \text{Carboxylation rate when RuBP regeneration is limited by electron transport} \]
\[ \min(W_c, W_j) = \text{Minimum value between these two rate variables} \]
\[ \Gamma = \text{CO}_2 \text{ compensation point in the absence of dark respiration} \]
\[ C_i = \text{Internal CO}_2 \text{ concentration} \]

Internal CO\(_2\) concentration can be expressed as
\[
C_i = C_s - \frac{A}{g_s} \tag{69}
\]

Leaf surface CO\(_2\) concentration, \(C_s\), can be expressed as
\[
C_s = C_a - \frac{A}{g_b} \tag{70}
\]

where
$C_a = \text{Atmosphere’s CO}_2 \text{ concentration } (= 360 \text{ ppm})$

$g_b = \text{Conductance across the laminar boundary layer of a leaf (mol m}^{-2}\text{ s}^{-1}) \text{ for CO}_2 \text{ exchange}$

$$g_b = \frac{1}{R_a+R_{b,CO_2}} \quad (71)$$

Both, $W_j$ and $W_c$ take the algebraic form

$$V_c - 0.5V_o = \frac{aC_i-ad}{eC_i+b} \quad (72)$$

CO$_2$ compensation point in the absence of dark respiration, $\Gamma$ can be calculated as

$$\Gamma = \frac{0.105\cdot K_c \cdot [O_2]}{K_o} \quad (73)$$

where

$K_c = \text{Michaelis-Menten coefficients for CO}_2 \ (= 333 \text{ microbars at 25 °C})$

$K_o = \text{Michaelis-Menten coefficients for O}_2 \ (= 295 \text{ millibars at 25 °C})$

$[O_2] = \text{Partial pressures of O}_2 \text{ in the intercellular air space } (= 210 \mu\text{mol/mol})$

2.11.5. Derivation of photosynthetic flux density of a leaf

The goal is to derive an equation describing $A$ that is independent of $C_s$, $C_i$, and $g_s$, the term $C_s$ is eliminated by inserting Equation 74 into Equations 73 and 69.

$$C_i = C_a - \frac{A}{g_b} - \frac{A}{g_s} \quad (74)$$

$$g_s = \frac{mA rh}{C_a \cdot A} + b' \quad (75)$$

The term, $g_s$ is eliminated by inserting Equation 79 to Equation 78.
\[
C_i = C_a - \frac{A}{g_b} - \frac{A}{mArh + b'} = C_a - \frac{A}{g_b} - \frac{A(C_a - \frac{A}{g_b})}{mArh + b'(C_a - \frac{A}{g_b})}
\]

\[
= \frac{C_a g_b [mArh + b'(C_a - \frac{A}{g_b})] - A [mArh + b'(C_a - \frac{A}{g_b})] - A g_b (C_a - \frac{A}{g_b})}{g_b [mArh + b'(C_a - \frac{A}{g_b})]}
\]

\[
= \frac{C_a g_b mArh + C_a^2 b' g_b - C_a b'A - mrhA^2 - Ab'C_a + \frac{b'A^2}{g_b} - g_b AC_a + A^2}{g_b mArh + b'C_ag_b - b'A}
\]

\[
= \frac{(1 + \frac{b'}{g_b} - mrh)A^2 + C_a (g_b mrh - 2b' - g_b) A + C_a^2 b' g_b}{(g_b mrh - b')A + b'C_ag_b} = \frac{\alpha A^2 + \beta A + \gamma}{\theta A + \frac{\gamma}{C_a}}
\]

(76)

\[
\alpha = 1 + \frac{b'}{g_b} - mrh
\]

(77)

\[
\beta = C_a (g_b mrh - 2b' - g_b)
\]

(78)

\[
\gamma = C_a^2 b' g_b
\]

(79)

\[
\theta = g_b mrh - b'
\]

(80)

From Equations 70 and 76,

\[
A + R_d = \frac{aC_i - ad}{eC_i + b}
\]

\[
AeC_i + Ab + R_d eC_i + R_d b = aC_i - ad
\]

\[
(Ae + R_d e - a)C_i = -ad - R_d b - Ab
\]

\[
C_i = \frac{-ad - R_d b - Ab}{Ae + R_d e - a}
\]

(81)
With Equations 80 and 85,

\[
\frac{\alpha A^2 + \beta A + \gamma}{\theta A + \frac{\gamma}{c_a}} = \frac{-ad-R_d b - Ab}{Ae + R_d e - a}
\]

\[
(aA^2 + \beta A + \gamma)(Ae + R_d e - a) = (-ad - R_d b - Ab) \left(\theta A + \frac{\gamma}{c_a}\right)
\]

\[
\therefore e\alpha A^3 + \alpha R_d e A^2 - a\alpha A^2 + e\beta A^2 + \beta R_d e A - a\beta A + e\gamma A + R_d e\gamma - a\gamma =
\]

\[-ad\theta A - ad \frac{\gamma}{c_a} - R_d b\theta A - R_d b \frac{\gamma}{c_a} - b\theta A^2 - b \frac{\gamma}{c_a} A
\]

\[
\therefore e\alpha A^3 + \left(e\beta + b\theta - a\alpha + \alpha R_d e\right) A^2 + \left(e\gamma + b \frac{\gamma}{c_a} - a\beta + ad\theta + \beta R_d e + R_d b \frac{\gamma}{c_a}\right) A - a\gamma + ad \frac{\gamma}{c_a} + R_d e\gamma + R_d b \frac{\gamma}{c_a} = 0
\]

\[
\therefore A^3 + \frac{e\beta + b\theta - a\alpha + \alpha R_d e}{e\alpha} A^2 + \frac{e\gamma + b \frac{\gamma}{c_a} - a\beta + ad\theta + \beta R_d e + R_d b \frac{\gamma}{c_a}}{e\alpha} A
\]

\[
+ \frac{-a\gamma + ad \frac{\gamma}{c_a} + R_d e\gamma + R_d b \frac{\gamma}{c_a}}{e\alpha} = 0
\]  

(82)

The solution of the cubic equation is taken from Press et al. (1989). If Equation 86 is manipulated into the form

\[
x^3 + px^2 + qx + r = 0
\]  

(83)

where

\[
p = \frac{e\beta + b\theta - a\alpha + eaR_d}{e\alpha}
\]

\[
q = \frac{e\gamma + b \frac{\gamma}{c_a} - a\beta + ad\theta + eR_d \beta + b\theta R_d}{e\alpha}
\]
\[ r = \frac{-ay + ad r}{c a} + e \gamma R_d + R_d b \frac{r}{c a} \]

three roots for the cubic equation are

\[ x_1 = -2\sqrt{Q}\cos\left(\frac{\theta}{3}\right) - \frac{p}{3} \]

\[ x_2 = -2\sqrt{Q}\cos\left(\frac{\theta + 2\pi}{3}\right) - \frac{p}{3} \]

\[ x_3 = -2\sqrt{Q}\cos\left(\frac{\theta + 4\pi}{3}\right) - \frac{p}{3} \]

where

\[ Q = \frac{p^2 - 3q}{9} \]

\[ R = \frac{2p^3 - 9pq + 27r}{54} \]

\[ \theta = a \cos\left(\frac{R}{\sqrt{Q^2}}\right) \]

The photosynthetic flux density of a leaf (A) corresponds to root number three \( (x_3) \). The variables \( a, b, d, \) and \( e \) are coefficient from Equation 76. If \( W_c \) is minimal, these coefficients correspond to

\[ W_c = V_c - 0.5V_o = \frac{ac_i - ad}{e\bar{c} + b} = \frac{V_{c max}(C_i - \Gamma)}{c_i + \Gamma (1 + [O_2])} \]

where

- \( V_{c max} \) = Maximum carboxylation rate when RuBP carboxylase/oxygenase is saturated
- \( K_c \) = Michaelis-Menten coefficients for CO\(_2\) (= 333 microbars at 25 °C)
- \( K_o \) = Michaelis-Menten coefficients for O\(_2\) (= 295 millibars at 25 °C)
- \( \Gamma \) = CO\(_2\) compensation point in the absence of dark respiration
- \([O_2]\) = Partial pressures of O\(_2\) in the intercellular air space (= 210 μmol/mol)
\[ V_{c\, max} = \frac{V_c(25^\circ C)\exp\left(\frac{(T-298)E}{298RT}\right)}{1+\exp\left(\frac{ST-H}{RT}\right)} \]  

(92)

\[ S = \text{constants for Boltzmann distribution temperature function (≈710)} \]

\[ H = \text{constants for Boltzmann distribution temperature function (≈220,000)} \]

The temperature dependencies of the \( K_c \) and \( K_o \) can be compensated for with Arrhenius Equation (Farquhar et al. 1980):

\[ K_c = K_c(25^\circ C)\exp\left(\frac{(T-298)E}{298RT}\right) \]  

(93)

\[ K_o = K_o(25^\circ C)\exp\left(\frac{(T-298)E}{298RT}\right) \]  

(94)

where

\[ E = \text{Relevant activation energy} \]

\[ R = \text{Universal gas constant} \]

\[ T = \text{Absolute leaf temperature} \]

If \( W_j \) is minimal, \( a, b, d, \) and \( e \) coefficients correspond to

\[ W_j = V_c - 0.5V_o = \frac{ac_i-ad}{ec_i+b} = \frac{J(c_i-I)}{4c_i+8I} \]  

(95)

where \( J \) is the potential rate of electron transport and expressed as (Harley et al. 1992):

for sunlit leaves in the \( j^{th} \) layer of the canopy

\[ J = \frac{\alpha PAR_{sun,j}}{\sqrt{\frac{a^2 PAR_{sun,j}^2}{J_{max}^2} + 1}} \]  

(96)

and, for shaded leaves in the \( j^{th} \) layer of the canopy

\[ J = \frac{\alpha PAR_{shade,j}}{\sqrt{\frac{a^2 PAR_{shade,j}^2}{J_{max}^2} + 1}} \]  

(97)
where

\[ \alpha = \text{Efficiency of light energy conversion on an incident light basis} = 0.22 \text{ mol electrons/mol photons} \]

\[ PAR_{sun,j} = \text{Flux density of PAR on sunlit leaves in the } j^{th} \text{ layer of canopy} \]

\[ PAR_{shade,j} = \text{Flux density of PAR on shaded leaves in the } j^{th} \text{ layer of canopy} \]

\[ J_{max} = \text{Light-saturated rate of electron transport} = 171 \text{ at 25 °C for trees} \]

The temperature dependencies of the \( J_{max} \) can be compensated for with Arrhenius Equation (Farquhar et al. 1980);

\[
J_{max} = \frac{J_{max}(25^\circ C) \exp \left( \frac{(T-298)E}{298RT} \right)}{1 + \exp \left( \frac{ST-H}{RT} \right)}
\]

where

\[ E = \text{Relevant activation energy} \]

\[ R = \text{Universal gas constant} \]

\[ T = \text{Absolute leaf temperature} \]

Table 4 summarizes CO\(_2\) exchange and stomatal conductance model parameters on sunlit/shaded leaves.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{cmax} ) (25°C)</td>
<td>( \mu \text{mol CO}_2 \text{ m}^{-2} \text{s}^{-1} )</td>
<td>90</td>
</tr>
<tr>
<td>( J_{max} ) (25°C)</td>
<td>( \mu \text{mol } (e^-) \text{ m}^{-2} \text{s}^{-1} )</td>
<td>171</td>
</tr>
<tr>
<td>( K_c ) (25°C)</td>
<td>Pa</td>
<td>33.3</td>
</tr>
<tr>
<td>( K_o ) (25°C)</td>
<td>kPa</td>
<td>29.5</td>
</tr>
<tr>
<td>Activation energy value for ( V_{cmax} )</td>
<td>J mol(^{-1})</td>
<td>64,637</td>
</tr>
<tr>
<td>Activation energy value for ( J_{max} )</td>
<td>J mol(^{-1})</td>
<td>37,000</td>
</tr>
<tr>
<td>Activation energy value for ( K_c )</td>
<td>J mol(^{-1})</td>
<td>65,120</td>
</tr>
<tr>
<td>Activation energy value for ( K_o )</td>
<td>J mol(^{-1})</td>
<td>13,990</td>
</tr>
<tr>
<td>Activation energy value for ( R_d )</td>
<td>J mol(^{-1})</td>
<td>51,176</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>( \text{mol e}^- \text{ (mol quanta)}^{-1} )</td>
<td>0.22</td>
</tr>
<tr>
<td>( m )</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>( b' )</td>
<td>( \text{mol m}^{-2} \text{s}^{-1} )</td>
<td>0.02</td>
</tr>
</tbody>
</table>
2.11.6. Weighted stomatal conductance for each layer

According to Norman (1982), stomatal conductance for sunlit and shaded leaves in a layer of the canopy can be weighted with sunlit and shaded leaf area index for that layer to estimate the stomatal conductance for that layer.

\[ g_{s,j} = \Delta F_j^* g_{s,sun,j} + (\Delta F - \Delta F_j^*) g_{s,shade,j} \]  
(99)

2.11.7. Stomatal conductance for whole layers

Stomatal conductance for the whole layers can be calculating by taking summation of the stomatal conductance for each layer. Stomatal resistance is a reciprocal of stomatal conductance.

\[ g_s = \sum_{j=1}^{N} g_{s,j} \]  
(100)

2.12. Transpiration flux calculation

Transpiration is the escape of water vapor from plants as controlled to a considerable degree by leaf resistances. The process is comprised of two stages: evaporation of water from cell walls and diffusion out of the leaf mainly through stomata (Kramer 1983). Transpiration flux, \( T_f (g \text{ m}^{-2} \text{ hr}^{-1}) \) is estimated as (Kramer 1983):

\[ T_f = \frac{C_{leaf} - C_{air}}{\frac{1}{g_s} + R_a} \cdot \frac{3600}{LAI} \]  
(101)

where

- \( C_{leaf} \) = Water vapor concentration at the evaporating surfaces within the leaf (g m\(^{-3}\))
- \( C_{air} \) = Water vapor concentration in the air (g m\(^{-3}\))
- \( g_s \) = Stomatal conductance (s m\(^{-1}\))
- \( R_a \) = Aerodynamic resistance (s m\(^{-1}\))
- \( LAI \) = Leaf area index

\( C_{leaf} \) and \( C_{air} \) are calculated as (Monteith and Unsworth 1990):

\[ C_{leaf} = \frac{M_w \varepsilon_s}{RT} = \frac{2165 \varepsilon_s}{T} \]  
(102)
\[ C_{\text{air}} = \frac{M_{w}e}{RT} = \frac{2165e}{T} \]  

\[ M_w = \text{Molecular weight of water (}=18 \text{ g mol}^{-1}=18000 \text{ g kmol}^{-1}) \]

\[ R = \text{Universal gas constant (}=8.314 \text{ J mol}^{-1} \text{ K}^{-1}=8.314 \text{ kPa m}^{-3} \text{ kmol}^{-1} \text{ K}^{-1}) \]

\[ e_s = \text{Saturation vapor pressure (kPa)} \]

\[ e = \text{Vapor pressure (kPa)} \]

\[ T = \text{Temperature (K)} \]

### 2.13. Air quality improvement calculation

Hourly air quality improvement per unit tree cover due to the dry deposition of air pollutants, \( I_{\text{unit}} \) (\%) is calculated as:

\[ I_{\text{unit}} = \frac{F}{F+M_{\text{total}}} \cdot 100 \]  

where

\[ F = \text{Pollutant flux (g m}^{-2} \text{ h}^{-1}) \]

\[ M_{\text{total}} = \text{Total air pollutant mass per unit tree cover (g m}^{-2} \text{ h}^{-1}) \]

\[ M_{\text{total}} = H \cdot C \]  

where

\[ H = \text{Urban mixing height (m)} \]

\[ C = \text{Air pollutant concentration (g m}^{-3} \text{ h}^{-1}) \]

For PM2.5, if the net flux (F) is negative

\[ I_{\text{unit}} = \frac{F}{M_{\text{total}}} \cdot 100 \]  

Hourly air quality improvement for total tree cover, \( I_{\text{total}} \) (\%) is calculated as:

\[ I_{\text{total}} = \frac{F \cdot \frac{T_c}{100}}{F \cdot \frac{T_c}{100}+M_{\text{total}}} \cdot 100 \]  

where
\[ T_c = \text{Total tree cover in the city (\%)} \]

For PM2.5, if \( F = -M_{\text{total}} \) and \( T_c = 100 \), the denominator in equation 111 becomes 0 and \( I_{\text{total}} \) becomes infinity. To avoid this, the equation below should be used.

\[ I_{\text{total}} = I_{\text{unit}} \cdot \frac{T_c}{100} \]  
(108)

### 2.14. Air pollutant concentration change calculation

Change in air pollutant concentration can be calculated as

\[ \Delta C = \frac{C}{1-I_{\text{total}}/100} - C \]  
(109)

where

\[ \Delta C = \text{Air pollutant concentration change (ppm for CO, NO}_2, \text{SO}_2, \text{and O}_3 \text{and } \mu g \text{ m}^{-3} \text{ for PM10 and PM2.5)} \]

\[ C = \text{Air pollutant concentration (ppm for CO, NO}_2, \text{SO}_2, \text{and O}_3 \text{and } \mu g \text{ m}^{-3} \text{ for PM10 and PM2.5)} \]

### 2.15. Monetary value calculation

Monetary value of pollution removal by trees is estimated using the median externality values for the United States for each pollutant (Murray et al. 1994, Ottinger et al. 1990) adjusted to 2007 dollars based on the producer price index (US Dept. of Labor 2008). The externality values are: CO=$1,407t^{-1}$, NO$_2$=$9,906t^{-1}$, PM10=$6,614t^{-1}$, SO$_2$=$2,425t^{-1}$. Externality value for O$_3$ was set to equal the value for NO$_2$.

### 3. References


