STRATUM Population Estimators and Standard Error Equations for Simple Random Street Segment Sampling:

Sample size (number of street segments) = \( n \)

Sample size of k-th subpopulation = \( n_k \)

Value measured at street segment \( i \) is \( y_i \) (with \( i = 1, 2, \ldots, n \) )

Value measured at street segment \( i \) within subpopulation \( k \): \( y_{ki} \) (with \( i = 1, 2, \ldots, n_k \) )

Population size (number of street segments) = \( N \)

Citywide sample mean = \( \bar{y} = \frac{1}{n} (y_1 + y_2 + \cdots + y_n) \) \hspace{1cm} \text{Equation 1}

Estimate of the citywide population total = \( \hat{\tau} = N\bar{y} \) \hspace{1cm} \text{Equation 2}

Citywide sample variance = \( s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \bar{y})^2 \) \hspace{1cm} \text{Equation 3}

Estimated variance of the citywide sample mean = \( \hat{v}(\bar{y}) = \left( \frac{N-n}{N} \right) \frac{s^2}{n} \) \hspace{1cm} \text{Equation 4}

Standard error (se) of the citywide sample mean = \( \sqrt{\hat{v}(\bar{y})} \) \hspace{1cm} \text{Equation 5}

Estimated variance of the estimate of the citywide total = \( \hat{v}(\hat{\tau}) = N^2 \hat{v}(\bar{y}) \) \hspace{1cm} \text{Equation 6}

Standard error (se) of the estimated citywide total = \( \sqrt{\hat{v}(\hat{\tau})} \) \hspace{1cm} \text{Equation 7}

Estimate of the subpopulation (zone) total = \( \hat{\tau}_k = \frac{N_k}{n_k} \sum_{i=1}^{n_k} y_{ki} \) \hspace{1cm} \text{Equation 8}

Subpopulation (zone) sample variance = \( s_k^2 = \frac{1}{n_k-1} \sum_{i=1}^{n_k} (y_{ki} - \bar{y}_k)^2 \) \hspace{1cm} \text{Equation 9}

Estimated variance of the estimate of the subpopulation (zone) total = \( \hat{v}(\hat{\tau}_k) = N_k^2 \left( \frac{N_k-n_k}{N_kn_k} \right) s_k^2 \) \hspace{1cm} \text{Equation 10}

Standard error (se) of the estimated subpopulation (zone) total = \( \sqrt{\hat{v}(\hat{\tau}_k)} \) \hspace{1cm} \text{Equation 11}

Example (1) citywide calculations:
The city of Evergreen possesses 100 total street segments (\( N \)). For purposes of a STRATUM analysis, a 4\% random sample of street segments were inventoried for municipal street trees. The number of trees in the 4 (\( n \)) sample units (\( y_1, y_2, y_3, y_4 \)) were 11, 9, 12, and 7, respectively.

The mean number of trees per segment (using Equation 1) is
\[ \bar{y} = \left( \frac{11 + 9 + 12 + 7}{4} \right) = 9.75 \]

The sample variance (using Equation 3) is
\[ s^2 = \frac{(11 - 9.75)^2 + (9 - 9.75)^2 + (12 - 9.75)^2 + (7 - 9.75)^2}{4 - 1} = 4.9166 \]

The estimated variance of the sample mean (using Equation 4) is
\[ \hat{\nu}(\bar{y}) = \left( \frac{100 - 4}{100} \right) \frac{4.9166}{4} = 1.1799 \]
so that the estimated standard error (using Equation 5) is \( \sqrt{1.1799} = 1.086 \).

An estimate of the total number of trees in the city (using Equation 2) is
\[ \hat{\tau} = 100(9.75) = 975 \]
The estimated variance associated with the estimate of the total (using Equation 6) is
\[ \hat{\nu}(\hat{\tau}) = 100^2 \times 1.1799 = 11799 \]
giving an estimated citywide standard error (using Equation 7) of \( \sqrt{11799} = 108.62 \). The city of Evergreen tree population, therefore, is 975 ±109 (estimate ± standard error).

Example (2) zone calculations:
Following Example 1, the city of Evergreen possesses 100 total street segments (\( N \)). For purposes of a STRATUM analysis, a 4% random sample of street segments were inventoried for municipal street trees. The number of trees in the 4 (\( n \)) sample units (\( y_1, y_2, y_3, y_4 \)) were 11, 9, 12, and 7, respectively. The city is divided into two management zones (\( k = 1,2 \)), where \( k = 1 \) includes sample units \( y_1 \) and \( y_2 \) and \( k = 2 \) includes sample units \( y_3 \) and \( y_4 \). The total number of segments per zone (\( k_{1,2} \)) is 40 and 60, respectively.

An estimate of the total number of trees in the management zone 1 (using Equation 8) is
\[ \hat{\tau}_k = \frac{40}{2} (11 + 9) = 400 \]
The sample variance of the sampling units associated with zone 1 (using Equation 9) is
\[ s_k^2 = \frac{1}{2-1} \left( (11-10)^2 + (9-10)^2 \right) = 1 \]
The estimated variance associated with the estimate of the zone 1 total (using Equation 10) is
\[ \hat{\nu}(\hat{\tau}_k) = 40^2 \left( \frac{40 - 2}{40(2)} \right)(1) = 760 \]
giving an estimated zone 1 standard error (using Equation 11) of \( \sqrt{760} = 27.57 \). The zone 1 tree population, therefore, is 400 ±28 (estimate ± standard error).

Notice that the city of Evergreen zone 2 tree population would equal 570 ±147, where Example 2 equations were used. The citywide population estimation is, therefore, additive (zone1 [400] and zone 2 [570] sum to a total of 970). However, the standard error citywide does not equal the sum of the standard error by zone (28+147≠109).