

i-Tree Eco Dry Deposition Model Descriptions

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1. Introduction

Employing field-surveyed urban forest information, location specific data, weather data, and air pollutant measurements, i-Tree Eco assesses the structure of community trees and quantifies the environmental services that trees provide. i-Tree Eco was developed based on the Urban Forest Effects (UFORE) model. With its dry deposition component (UFORE-D) integrated into i-Tree Eco, dry deposition of air pollution (i.e., pollution removal during nonprecipitation periods) to trees and shrubs and associated percent improvement in air quality can be estimated with i-Tree Eco. The dry deposition of criteria air pollutants (CAPs); carbon monoxide (CO), nitrogen dioxide (NO₂), ozone (O₃), sulfur dioxide (SO₂), and particulate matter less than 10 microns (PM10) can be assessed with version 4. In addition, particulate matter less than 2.5 microns (PM2.5) can be assessed with version 5. This document provides detailed i-Tree Eco dry deposition model descriptions.

2. Model Descriptions

i-Tree Eco provides functions to calculate values listed below. These functions are described in this section.

- Air pollutant flux calculation
- Deposition velocity calculation
- Aerodynamic resistance calculation
- Friction velocity calculation
- Quasi-laminar boundary layer resistance calculation
- Canopy resistance calculation
- Stomatal resistance calculation
- Transpiration calculation
- Air quality improvement calculation
- Air pollutant concentration change calculation
- Monetary value calculation

2.1. Air pollutant flux calculation for CO, NO₂, SO₂, O₃ and PM10

Pollutant flux is calculated as a product of the deposition velocity and the air pollutant

concentration:

$$F = V_d \cdot C \cdot 3600 \quad (1)$$

$$F_{min} = V_{d,min} \cdot C \cdot 3600 \quad (2)$$

$$F_{max} = V_{d,max} \cdot C \cdot 3600 \quad (3)$$

where

F	=	Pollutant flux ($\text{g m}^{-2} \text{h}^{-1}$)
F_{min}	=	Minimum pollutant flux ($\text{g m}^{-2} \text{h}^{-1}$)
F_{max}	=	Maximum pollutant flux ($\text{g m}^{-2} \text{h}^{-1}$)
V_d	=	Deposition velocity (m s^{-1})
$V_{d,min}$	=	Minimum deposition velocity (m s^{-1})
$V_{d,max}$	=	Maximum deposition velocity (m s^{-1})
C	=	Air pollutant concentration (g m^{-3})

2.2. Deposition velocity calculation for CO, NO₂, SO₂, and O₃

Deposition velocities (V_d) for CO, NO₂, SO₂, and O₃ are calculated as the inverse of the sum of the aerodynamic resistance (R_a), quasi-laminar boundary layer resistance (R_b), and canopy resistance (R_c) (Balducchi et al. 1987). The aerodynamic resistance can be calculated with meteorological data and thus is independent of the air pollutant types, while the quasi-laminar boundary layer and canopy resistances are separately calculated for CO, NO₂, O₃, and SO₂. In addition, the canopy resistance is calculated depending on in-leaf and out-of-leaf periods.

$$V_d = \frac{1}{R_a + R_b + R_c} \quad (4)$$

where

V_d	=	Deposition velocity (m s^{-1})
R_a	=	Aerodynamic resistance (s m^{-1})
R_b	=	Quasi-laminar boundary layer resistance for a type of air pollutions (s m^{-1})
R_c	=	Canopy resistance (s m^{-1})

Depending on the flux density of photosynthetically active radiation (PAR), minimum and maximum of V_d are estimated as follows (Lovett 1994):

If PAR > 0 (in the daytime)

for NO₂

$$V_{d,min} = 0.001$$

$$V_{d,min} = 0.005 \quad (5)$$

for O₃

$$V_{d,min} = 0.001$$

$$V_{d,min} = 0.008 \quad (6)$$

for SO₂

$$V_{d,min} = 0.002$$

$$V_{d,min} = 0.01 \quad (7)$$

else (in the nighttime)

$$V_{d,min} = V_d$$

$$V_{d,min} = V_d \quad (8)$$

2.3. Deposition velocity calculation for PM10

Deposition velocity for PM10 is calculated based on average, minimum, and maximum values reported by Lovett (1994).

$$V_d = V_{d,PM10,avg} \cdot \frac{BAI+LAI}{BAI+LAI_{PM10}} \quad (9)$$

$$V_{d,min} = V_{d,PM10,min} \cdot \frac{BAI+LAI}{BAI+LAI_{PM10}} \quad (10)$$

$$V_{d,max} = V_{d,PM10,max} \cdot \frac{BAI+LAI}{BAI+LAI_{PM10}} \quad (11)$$

where

- $V_{d,PM10,avg}$ = Average deposition velocity for PM10 (= 0.0064 ms⁻¹) (Lovett 1994)
 $V_{d,PM10,min}$ = Minimum deposition velocity for PM10 (= 0.0025 ms⁻¹) (Lovett 1994)
 $V_{d,PM10,max}$ = Maximum deposition velocity for PM10 (= 0.01 ms⁻¹) (Lovett 1994)
 LAI_{PM10} = Leaf area index for particle deposition (= 6)
 BAI = Bark area index
 LAI = Leaf area index

2.4. Deposition velocity calculation for PM2.5

Deposition velocities of PM2.5 to trees were estimated from literature and varied with wind speed (Beckett et al 2000, Freer-Smith et al. 2004, Pullman 2009). These papers measured deposition velocities to tree leaves from 17 tree species under wind speeds of 1, 3, 6, 8, 9 and 10 m s⁻¹. For each wind speed, the median deposition velocities from the measured deposition velocities was used to estimate the $V_{d,PM2.5}$ for that wind speed per unit leaf area (Table 1).

$$V_d = V_{d,PM2.5} \cdot LAI \quad (12)$$

The standard error of the estimates among the species was used to estimate a potential range of values of deposition velocities. The median deposition velocity per wind speed plus 1.96 times the standard error was used to estimate a maximum deposition for the wind speed ($V_{d,PM2.5,Max}$). As using standard errors to estimate the lower range of deposition velocities produced negative deposition velocities, the minimum average V_d from any species was used to represent the minimum V_d for the wind speed ($V_{d,PM2.5,Min}$). To estimate the V_d for wind speeds between 1 and 10 m s⁻¹ that did not have a measured V_d , values were interpolated between the closest measured values. For wind speeds above 10 m s⁻¹, the V_d for 10 m s⁻¹ was used; for a wind speed of 0 m s⁻¹, the V_d was assumed to be 0 cm s⁻¹ (Table 2).

$$V_{d,min} = V_{d,PM2.5,min} \cdot LAI \quad (13)$$

$$V_{d,max} = V_{d,PM2.5,max} \cdot LAI \quad (14)$$

Table 1 Summary of average deposition velocities (cm/sec) of PM2.5 by wind speed from the literature per unit leaf area

Species	Wind Speed (m s ⁻¹)				
	1	3	6	8.5 ^a	10

<i>Quercus petraea</i> ^b		0.831	1.757	3.134	
<i>Alnus glutinosa</i> ^b		0.125	0.173	0.798	
<i>Fraxinus excelsior</i> ^b		0.178	0.383	0.725	
<i>Acer pseudoplatanus</i> ^b		0.042	0.197	0.344	
<i>Psuedotsuga menziesii</i> ^b		1.269	1.604	6.04	
<i>Eucalyptus globulus</i> ^b		0.018	0.029	0.082	
<i>Ficus nitida</i> ^b		0.041	0.098	0.234	
<i>Pinus nigra</i> ^c	0.13	1.15		19.24	28.05
<i>Cupressocyparis x leylandii</i> ^c	0.08	0.76		8.24	12.2
<i>Acer campestre</i> ^c	0.03	0.08		0.46	0.57
<i>Sorbus intermedia</i> ^c	0.04	0.39		1.82	2.11
<i>Populus deltoides</i> ^c	0.03	0.12		1.05	1.18
<i>Pinus strobus</i> ^d	0.0108				
<i>Tsuga canadensis</i> ^d	0.0193				
<i>Tsuga japonica</i> ^d	0.0058				
<i>Picea abies</i> ^e	0.0189				
<i>Picea abies</i> ^e	0.038				
Median	0.030	0.152	0.197	0.924	2.110
SE ^f	0.012	0.133	0.281	1.610	5.257
Maximum ^g	0.057	0.442	0.862	5.063	14.542
Minimum ^h	0.006	0.018	0.029	0.082	0.570

^a combination of 8 and 9 m s⁻¹ wind speeds

^b from Freer-smith (2004)

^c from Beckett et al (2000)

^d from Pullman (2009). Included particles up to 3.0 µm in diameter

^e from Pullman (2009). Based on maximum and minimum of reported range. Included particles up to 3.8 µm in diameter.

^f standard error

^g based on median plus one standard error

^h based on lowest recorded value for any species

Resuspension of PM_{2.5} from trees were estimated from Pullman (2009) and varied with wind speed. This paper measured percent resuspension of PM_{2.5} from tree leaves of three tree species under wind speeds of 6.5, 10 and 13 m s⁻¹. The average percent resuspension for the trees species

and wind speed was calculated (Table 2). As the percent resuspension for the wind speed of 6.5 m s⁻¹ was 9.5%, a value of 9% was assumed for a wind speed of 6 m s⁻¹ and 10% for 7 m s⁻¹. The percent resuspension for a wind speed of 0 m s⁻¹ was assumed to be 0%. To estimate the percent resuspension for wind speeds between 0 and 13 m s⁻¹ that did not have a measured resuspension rates, values were interpolated between the closest measured values (or assumed value at wind speed of 0 m s⁻¹). For wind speeds above 13 m s⁻¹, the percent resuspension rate for 13 m s⁻¹ was used (Table 2).

Table 2 Deposition velocities and percent resuspension by wind speed per unit leaf area

Wind speed (m s ⁻¹)	Deposition Velocity (V_d)			Resuspension (%)
	Average (cm s ⁻¹)	Minimum (cm s ⁻¹)	Maximum (cm s ⁻¹)	
0	0.00	0.000	0.000	0.0
1	0.03	0.006	0.042	1.5
2	0.09	0.012	0.163	3.0
3	0.15	0.018	0.285	4.5
4	0.17	0.022	0.349	6.0
5	0.19	0.025	0.414	7.5
6	0.20	0.029	0.478	9.0
7	0.56	0.056	1.506	10.0
8	0.92	0.082	2.534	11.0
9	0.92	0.082	2.534	12.0
10	2.11	0.570	7.367	13.0
11	2.11	0.570	7.367	16.0
12	2.11	0.570	7.367	20.0
13	2.11	0.570	7.367	23.0

2.5. Air pollutant flux calculation for PM2.5

During non-precipitation periods, flux values of PM2.5 are cumulated on leaves hourly with a percent of the accumulated PM2.5 resuspended back to the atmosphere based on local wind speed. At time t , newly deposited PM2.5 flux can be calculated as

$$f_t = V_{d,PM2.5,t} \cdot C_t \cdot 3600 \quad (15)$$

$$f_{min,t} = V_{d,PM2.5,min,t} \cdot C_t \cdot 3600 \quad (16)$$

$$f_{max,t} = V_{d,PM2.5,max,t} \cdot C_t \cdot 3600 \quad (17)$$

where

f_t	=	PM2.5 flux at time t ($\text{g m}^{-2} \text{h}^{-1}$)
f_{min}	=	Minimum PM2.5 flux at time t ($\text{g m}^{-2} \text{h}^{-1}$)
f_{max}	=	Maximum PM2.5 flux at time t ($\text{g m}^{-2} \text{h}^{-1}$)
$V_{d,PM2.5,t}$	=	Deposition velocity at time t (m s^{-1})
$V_{d,PM2.5,min,t}$	=	Minimum deposition velocity at time t (m s^{-1})
$V_{d,PM2.5,max,t}$	=	Maximum deposition velocity at time t (m s^{-1})
$C_{adj,t}$	=	Air pollutant concentration at time t (g m^{-3})

Resuspended amount of PM2.5 at time t can be calculated as

$$R_t = (A_{t-1} + f_t) \cdot \frac{rr_t}{100} \quad (18)$$

$$R_{min,t} = (A_{min,t-1} + f_{min,t}) \cdot \frac{rr_t}{100} \quad (19)$$

$$R_{max,t} = (A_{max,t-1} + f_{max,t}) \cdot \frac{rr_t}{100} \quad (20)$$

where

R_t	=	PM2.5 flux resuspended to atmosphere at time t ($\text{g m}^{-2} \text{h}^{-1}$)
R_{min}	=	Minimum PM2.5 flux resuspended to atmosphere at time t ($\text{g m}^{-2} \text{h}^{-1}$)
R_{max}	=	Maximum PM2.5 flux resuspended to atmosphere at time t ($\text{g m}^{-2} \text{h}^{-1}$)
A_{t-1}	=	PM2.5 accumulated on leaves at time t ($\text{g m}^{-2} \text{h}^{-1}$)
$A_{min,t-1}$	=	Minimum PM2.5 accumulated on leaves at time t ($\text{g m}^{-2} \text{h}^{-1}$)
$A_{max,t-1}$	=	Maximum PM2.5 accumulated on leaves at time t ($\text{g m}^{-2} \text{h}^{-1}$)
rr_t	=	APM2.5 resuspension rate at time t (%)

After PM2.5 resuspended back to the atmosphere, accumulated amount on leaves at time t can be calculated as

$$A_t = (A_{t-1} + f_t) - R_t \quad (21)$$

$$A_{min,t} = (A_{min,t-1} + f_{min,t}) - R_{min,t} \quad (22)$$

$$A_{max,t} = (A_{max,t-1} + f_{max,t}) - R_{max,t} \quad (23)$$

After PM2.5 resuspended back to the atmosphere, net PM2.5 flux at time t can be calculated as

$$F_t = f_t - R_t \quad (24)$$

$$F_{min,t} = f_{min,t} - R_{min,t} \quad (25)$$

$$F_{max,t} = f_{max,t} - R_{max,t} \quad (26)$$

where

$$\begin{aligned} F_t &= \text{Net PM2.5 flux at time } t \text{ (g m}^{-2} \text{ h}^{-1}\text{)} \\ F_{min} &= \text{Minimum net PM2.5 flux at time } t \text{ (g m}^{-2} \text{ h}^{-1}\text{)} \\ F_{max} &= \text{Maximum net PM2.5 flux at time } t \text{ (g m}^{-2} \text{ h}^{-1}\text{)} \end{aligned}$$

When the net flux is negative due to a large amount of resuspension and the absolute value of the net flux is greater than the total PM2.5 amount in the atmosphere ($M_{total} \text{ (g m}^{-2} \text{ h}^{-1}\text{)} = C \text{ (g m}^{-3} \text{ h}^{-1}\text{)} \times \text{mixing layer height (m)}$), the net flux is adjusted to be equal to $-M \text{ (g m}^{-2} \text{ h}^{-1}\text{)}$.

During precipitation events, the accumulated PM2.5 is assumed to be washed off to the ground surface, depending upon the precipitation intercepted and stored on leaves and running off from leaves. Until it reaches its maximum capacity of storage ($0.2\text{mm} \times \text{LAI}$) (Wang et al. 2008, Hirabayashi 2012), precipitation is stored on leaves and no water runoffs from leaves. During this period, no PM2.5 is washed off. Once it reaches the maximum, water on leaves starts running off, with which all of the accumulated PM2.5 are assumed to be washed off. Thus, resuspension, accumulation and net flux for these time periods become 0.

2.6. Aerodynamic resistance calculation

The aerodynamic resistance (R_a) is calculated as (Killus et al. 1984):

$$R_a = \frac{u(z)}{u_*^2} \quad (27)$$

where

$$\begin{aligned} u(z) &= \text{Mean wind speed at height } z \text{ (ms}^{-1}\text{)} \\ u_* &= \text{Friction velocity (ms}^{-1}\text{)} \end{aligned}$$

2.7. Friction velocity calculation

Depending on the stability of atmosphere (i.e., neutral, unstable, and stable), u_* is calculated as described in the following sections. The stability of atmosphere can be determined by Monin-Obuhkov stability length, which can be empirically estimated based on the Pasquill stability class.

$$\text{Pasquill = A: } 1/L = -0.0875 \cdot z_0^{-0.1029}$$

$$\text{Pasquill = B: } 1/L = -0.03849 \cdot z_0^{-0.1714}$$

$$\text{Pasquill = C: } 1/L = -0.0807 \cdot z_0^{-0.3049}$$

$$\text{Pasquill = D: } 1/L = 0 \cdot z_0^{-0}$$

$$\text{Pasquill = E: } 1/L = 0.0807 \cdot z_0^{-0.3049}$$

$$\text{Pasquill = F: } 1/L = 0.03849 \cdot z_0^{-0.1714} \quad (28)$$

where

$$\begin{aligned} L &= \text{Monin-Obuhkov stability length} \\ z_0 &= \text{Roughness length} \end{aligned}$$

2.7.1. Neutral atmosphere (L=0)

For the neutral atmosphere, u_* is calculated as:

$$u_* = \frac{k \cdot u(z-d)}{\ln\left(\frac{z-d}{z_0}\right)} \quad (29)$$

where

- k = von Karman constant (=0.41)
- $u(z)$ = Mean wind speed at height z (m s^{-1})
- z = Height of the weather station (m)
- d = Displacement height (m)
- z_o = Roughness length (m)

2.7.2. Unstable atmosphere ($L < 0$)

When the atmosphere is unstable such as during daytime when the air convection occurs, u_* is calculated as (Killus et al. 1984):

$$u_* = \frac{k \cdot u(z-d)}{\ln\left(\frac{z-d}{z_o}\right) - \psi_M\left(\frac{z-d}{L}\right) + \psi_M\left(\frac{z_o}{L}\right)} \quad (30)$$

where

- Ψ_M : Stability function for momentum
- L : Monin-Obuhkov stability length

Stability function for momentum (Ψ_M) is calculated as (van Ulden and Holtslag 1985):

$$\psi_M = 2 \ln\left(\frac{1+x}{2}\right) + \ln\left(\frac{1+x^2}{2}\right) - 2 \tan^{-1}(x) + \frac{\pi}{2} \quad (31)$$

where x is calculated as (Dyer and Bradley 1982):

$$x = \frac{1}{\left(1 - 28 \frac{z}{L}\right)^{0.25}} \quad (32)$$

2.7.3. Stable atmosphere ($L > 0$)

When the atmosphere is stable u_* is calculated as (Venkatram 1980):

$$u_* = C_{DN} \cdot u(z) \left[\frac{1}{2} + \frac{1}{2} \sqrt{1 - \left(\frac{2u_o}{\sqrt{C_{DN}} \cdot u(z)} \right)^2} \right] \quad (33)$$

where

- C_{DN} = Neutral drag coefficient (dimensionless)

C_{DN} is calculated as (US EPA 1995):

$$C_{DN} = \frac{k}{\ln\left(\frac{z}{z_o}\right)} \quad (34)$$

u_o is calculated as (US EPA 1995):

$$u_o = \sqrt{\frac{\beta_m z g \theta_*}{T}} \quad (35)$$

where

- β_m = Dimensionless constant (= 4.7)
- g = Acceleration due to gravity (= 9.81 ms⁻²)
- T = Air temperature (K)

θ_* is calculated as (US EPA 1995)

$$\theta_* = 0.09(1 - 0.5N^2) \quad (36)$$

where

- N = Fraction of opaque cloud cover

To obtain real-valued solutions for u_* , the following condition must hold

$$\frac{2u_o}{\sqrt{C_{DN} \cdot u(z)}} \leq 1 \quad (37)$$

Otherwise, the friction velocity (u_*) is calculated as (US EPA 1995):

$$u_* = u_{*cr} \frac{u(z)}{u_{cr}} \quad (38)$$

where u_{cr} and u_{*cr} are calculated as (US EPA 1995):

$$u_{cr} = \sqrt{\frac{4}{C_{DN}}} u_o \quad (39)$$

$$u_{*cr} = \frac{C_{DN} u_{cr}}{2} \quad (40)$$

2.8. Quasi-laminar boundary layer resistance calculation

Quasi-laminar boundary layer resistance (R_b) is calculated as (Pederson et al. 1995)

$$R_b = 2(Sc)^{\frac{2}{3}}(Pr)^{-\frac{2}{3}}(ku_*)^{-1} \quad (41)$$

where

Sc = Schmidt number

Pr = Prandtl number (= 0.72)

The Schmidt number is equal to 1 for O₃, 0.76 for CO, 0.98 for NO₂, and 1.15 for SO₂.

2.9. Canopy resistance calculation for CO

2.9.1. In-leaf periods

As removal of CO by vegetation is not directly related to transpiration, the canopy resistance for CO in in-leaf periods ($R_{c,InLeaf}$) is set to a constant based on data from Bidwell and Fraser (1972).

$$R_{c,InLeaf} = 50,000 \text{ (sm}^{-1}\text{)} \quad (42)$$

2.9.2. Out-of-leaf Periods

The canopy resistance for CO in out-of-leaf periods ($R_{c,OutLeaf}$) is set to a constant based on data from Bidwell and Fraser (1972).

$$R_{c,OutLeaf} = 1,000,000 \text{ (sm}^{-1}\text{)} \quad (43)$$

2.10. Canopy resistance calculation for NO₂, O₃, and SO₂

The canopy resistances for NO₂, O₃, and SO₂ (R_c) can be calculated as

$$\frac{1}{R_c} = \frac{1}{r_s+r_m} + \frac{1}{r_{soil}} + \frac{1}{r_t} \quad (44)$$

where

r_s = Stomatal resistance

r_m = Mesophyll resistance

r_{soil} = Soil resistance (= 2941 sm^{-1} in growing season, =2000 sm^{-1} otherwise)

r_t = Cuticular resistance

r_m and r_t for NO_2 , O_3 , and SO_2 are presented in Table 3.

Table 3 Parameter values

Parameter	NO_2	O_3	SO_2
r_m	100 (s m^{-1}) (Hosker and Lindberg 1982)	10 (s m^{-1}) (Hosker and Lindberg 1982)	0 (s m^{-1}) (Wesely 1989)
r_t	20,000 (s m^{-1}) (Wesely 1989)	10,000 (s m^{-1}) (Taylor et al. 1988, Lovett 1994)	8,000 (s m^{-1}) (Taylor et al. 1988, Lovett 1994)

2.11. Stomatal resistance calculation

The visible part (400 – 700 nm) or photosynthetically active radiation (PAR) of the solar spectrum is largely absorbed to drive photosynthesis, while the near-infrared part (700 – 3000 nm) is largely scattered by vegetations (Norman 1982). Direct beam and sky diffuse radiation are intercepted very differently by canopies (Norman 1982). Diffuse radiation is used much more efficiently than direct radiation by a canopy of photosynthesizing leaves (Weiss and Norman 1985). The stomatal resistance is linked to leaf photosynthesis (Baldochi 1994). Calculating canopy photosynthetic rates from leaf photosynthetic rates requires dividing the canopy into N layers, in which ΔF , the leaf area index in each layer, usually chosen as 0.1 or 0.2 (Norman 1980). It is preferable to compute stomatal resistance on the basis of the irradiance on both sunlit and shaded leaves and weighting these resistances according to the fraction of sunlit and shaded leaf area (Baldochi et al., 1987).

Therefore, derivation of stomatal conductance g_s , the inverse of stomatal resistance of a canopy involves these steps, which are explained in the following sections.

1. Divide solar radiation above the canopy into visible (PAR) and near-infrared portions (Section 2.11.1).
2. Divide PAR above the canopy into direct beam and diffuse radiation (Section 2.11.1).
3. Divide the canopy into N layers and calculate sunlit and shaded leaf areas in each layer of the

- canopy (Section 2.11.2).
4. Calculate flux density of PAR intercepted by sunlit and shaded leaves in each layer of the canopy (Section 2.11.3).
 5. Calculate stomatal conductance for sunlit and shaded leaves in each layer of the canopy (Sections 2.11.4 and 2.11.5).
 6. Weight stomatal conductance for each layer according to the fraction of sunlit and shaded leaf area (Section 2.11.6).
 7. Accumulate stomatal conductance for sunlit and shaded leaves throughout all layers to derive stomatal conductance for the whole canopy (Section 2.11.7).

2.11.1. Visible and near-infrared, direct and diffuse components of solar incident

Weiss and Norman (1985) obtained equations to predict direct and diffuse components of solar irradiance in both the visible (PAR) and near-infrared wave bands from measurements of only the total incoming solar radiation. Direct visible component (R_{DV}), diffuse visible component (R_{dV}), direct near-infrared component (R_{DN}), and diffuse near-infrared component (R_{dN}) of the solar radiation can be approximated by (Weiss and Norman 1985):

$$R_{DV} = 600 \left(-0.185 \cdot \frac{P}{P_0} \cdot m \right) \cos\theta \quad (45)$$

$$R_{dV} = 0.4(600 - R_{DV}) \cos\theta \quad (46)$$

$$R_{DN} = \left[720 \exp \left(-0.06 \frac{P}{P_0} m \right) - w \right] \cos\theta \quad (47)$$

$$R_{dN} = 0.6(720 - R_{DN} - w) \cos\theta \quad (48)$$

where

- | | | |
|-------|---|------------------------------------|
| E | = | Extinction coefficient (= 0.185) |
| P | = | Actual pressure (kPa) |
| P_0 | = | Sea level pressure (= 101.325 kPa) |
| m | = | Optical air mass |

With a flat approximation to the atmosphere, m can be defined as:

$$m = \frac{1}{\cos\theta} \quad (49)$$

θ = Solar zenith angle

w = Water absorption in the near infrared for 10 mm of precipitable water

w can be derived from

$$w = 1320 \text{antilog}_{10}[-1.1950 + 0.4459 \log_{10} m - 0.0345 (\log_{10} m)^2] \quad (50)$$

The fraction of the visible (or near-infrared) in the direct beam was estimated by analyzing the relationship between the fraction and the ratio of measured to potential total solar radiation, *RATIO* (Weiss and Norman 1985).

$$f_V = \frac{R_{DV}}{R_V} \left[1 - \left(\frac{A - \text{RATIO}}{B} \right)^{\frac{2}{3}} \right] \quad (51)$$

$$f_N = \frac{R_{DN}}{R_N} \left[1 - \left(\frac{C - \text{RATIO}}{D} \right)^{\frac{2}{3}} \right] \quad (52)$$

$$\text{RATIO} = \frac{R_T}{R_V + R_N} \quad (53)$$

where

R_T = Measurement of total incoming solar radiation

$R_V = R_{DV} + R_{dV}$

$R_N = R_{DN} + R_{dN}$

A , B , C , and D are 0.9, 0.7, 0.88, and 0.68, respectively. *RATIO* is never allowed to exceed A or C . If *RATIO* should exceed A or C , it is set equal to A or C .

It is assumed that 46% of the averaged hourly solar radiation is in the visible (Norman 1982). Norman (1982) states this percentage is essentially independent of zenith angle and cloudiness. Flux densities of direct and diffuse radiation of PAR on top of the canopy can be estimated as

$$\text{PAR}_{dir} = f_V (0.46 R_T) F_c \quad (54)$$

$$\text{PAR}_{diff} = (1 - f_V) (0.46 R_T) F_c \quad (55)$$

where

F_c = Conversion factor from $W\ m^{-2}$ to $\mu E\ m^{-2}\ s^{-1}$ (=4.6)

F_c was derived by converting energy of solar irradiance into photosynthetic photon flux density (PPFD), which is a measure of the number of photons in the visible spectrum range falling on a $1\ m^2$ area per second. PPFD is a measure of PAR. The energy carried by electromagnetic radiation is contained in the photons that travel as a wave. The energy in a photon varies with its frequency, according to the equation:

$$E = h\nu = \frac{hc}{\lambda} \quad (56)$$

E = Energy (J)

h = Planck's constant ($= 6.626 \times 10^{-34}\ J\ s^{-1}$)

ν = Frequency

c = Speed of light ($= 3.0 \times 10^8\ m\ s^{-1}$)

λ = Wavelength (m)

The conversion is conducted based on the center wavelength (=550 nm) of the visible part of the spectrum. As one photon in a certain wavelength has the energy calculated by Eq. 4-xx, the number of photons in 550 nm comprising one joule (J) can be calculated as 27.70×10^{17} . This number can be converted into as 4.59×10^{-6} moles where one mole = Avogadro's number = 6.02×10^{23} . Watt (W) is a unit of power that measures rate of flow of energy. One (W) is equivalent to one (J) of energy per second. Thus, one (W) of power from light at 550 nm would need to provide 27.70×10^{17} photons, or 4.6×10^{-6} moles per second. Another unit, Einstein (E) is introduced here, which is used in irradiance and in photochemistry. One einstein is defined as one mole of photons, regardless of their frequency. One (W) of power from light would need to provide 4.6×10^{-6} (E) or 4.6 (μE) per second. Therefore, irradiance in $W\ m^{-2}$ can be converted into PPFD in $\mu E\ m^{-2}\ s^{-1}$ by multiplying 4.6.

2.11.2. Canopy layers and sunlit/shaded leaf area index for each layer

The canopy is divided into N layers, in which the leaf area index in each layer (ΔF) chosen as 0.1 or 0.2, to calculate the PAR on sunlit and shaded leaves separately, assuming a mean angle between the direction of the sun and the plane of sunlit leaves and the spherical leaf angle distribution. This procedure is based on methods explained in Norman (1980)

The direct beam transmittance below layer j of the canopy with a spherical leaf distribution is

$$T_{B,j} = \exp\left(-\frac{F_j}{2\cos\theta}\right) \quad (57)$$

where

$$\begin{aligned} F_j &= \text{Leaf area index at the layer } j \\ \theta &= \text{Zenith angle of the sun} \end{aligned}$$

For a spherical leaf distribution, the sunlit leaf area index in the j^{th} layer is given by ΔF_j^*

$$\Delta F_j^* = (T_{B,j} - T_{B,j+1})2\cos\theta \quad (58)$$

The fraction of leaf area in the j^{th} layer that is sunlit is given by

$$\frac{\Delta F_j^*}{\Delta F} \quad (59)$$

the shaded leaf area index in the j^{th} layer is given by

$$\Delta F - \Delta F_j^* \quad (60)$$

2.11.3. Flux density of PAR on sunlit/shaded leaves in each layer

Flux density of PAR on sunlit and shaded leaves can be estimated with methods outlined in Norman (1982) and Baldocchi et al. (1987). Shaded leaves receive only diffuse light. The flux density of diffuse PAR on shaded leaves in the j^{th} layer can be estimated as

$$PAR_{shade,j} = PAR_{diff} \exp(-0.5F^{0.7}) \cdot S_j + C_j \quad (61)$$

$$\begin{aligned} PAR_{diff} &= \text{Flux density of diffuse PAR above the canopy} \\ F &= \text{Leaf area index for whole canopy} \\ S_j &= \text{Scaling factor of diffuse flux density of PAR for the } j^{\text{th}} \text{ layer} \end{aligned}$$

$$S_j = \frac{\exp\left(-0.5F \cdot \frac{j-0.5}{N}\right)}{\sum_{j=1}^N \exp\left(-0.5F \cdot \frac{j-0.5}{N}\right)} \quad (62)$$

C arises from multiple scattering of direct beam radiation and is given by

$$C_j = 0.07PAR_{dir} \left[1.1 - 0.1 \left(F_j - \frac{\Delta F}{2}\right)\right] \exp(-\sin\beta) \quad (63)$$

Conceptually, the 0.07 represents a scattering coefficient, the term in brackets accounts for the decrease in multiple scattering with depth and the last exponential term accounts for the increased scattering at high zenith angles.

Sunlit leaves receive both direct and diffuse lights. The flux density of PAR on sunlit leaves in the j^{th} layer can be estimated as

$$PAR_{sun,j} = PAR_{dir} \frac{\cos\alpha}{\sin\beta} + PAR_{shade,j} \quad (64)$$

where

- PAR_{dir} = Flux density of direct PAR above the canopy
- α = Angle between a leaf and the sun (= 60 degrees)
- β = Solar elevation angle

$PAR_{dir}/\cos\beta$ is the visible irradiance on a plane perpendicular to the direction of the sun.

2.11.4. Stomatal conductance for sunlit/shaded leaves in each layer

This procedure is based on methods explained in Farquhar et al. (1980), Baldochi (1994), and Harley et al. (1992). Stomatal conductance g_s for each layer of the canopy can be computed as:

$$g_s = \frac{mArh}{C_s} + b' \quad (65)$$

where

- m = Dimensionless slope (= 10)
- A = Photosynthetic flux density of a leaf
- rh = Relative humidity
- b' = Zero intercept when A is equal to or less than zero (= $0.02 \text{ mol m}^{-2} \text{ s}^{-1}$)
- C_s = Leaf surface CO_2 concentration

A is represented as:

$$A = V_c - 0.5V_o - R_d \quad (66)$$

where

- V_c = Carboxylation rate of CO_2 exchange between leaf and atmosphere
- V_o = Oxygenation rate of CO_2 exchange between leaf and atmosphere

R_d = Dark respiration rate of CO₂ exchange between leaf and atmosphere

Dark respiration rate of CO₂ exchange between leaf and atmosphere, R_d can be calculated as

$$R_d = \frac{V_c(25^\circ\text{C}) \cdot 0.015 \exp\left[\frac{(T-298)E}{298RT}\right]}{1 + \exp[1.3(T-328)]} \quad (67)$$

$V_c(25^\circ\text{C})$ = Carboxylation rate of CO₂ exchange between leaf and atmosphere (=90)

E = Relevant activation energy

R = Universal gas constant

T = Absolute leaf temperature

The term $V_c - 0.5V_o$ is expressed by Baldocchi (1994) as

$$V_c - 0.5V_o = \min(W_c, W_j) \left(1 - \frac{\Gamma}{C_i}\right) \quad (68)$$

where

W_c = Carboxylation rate when ribulose bisphosphate (RuBP) carboxylase/oxygenase is saturated

W_j = Carboxylation rate when RuBP regeneration is limited by electron transport

$\min(W_c, W_j)$ = Minimum value between these two rate variables

Γ = CO₂ compensation point in the absence of dark respiration

C_i = Internal CO₂ concentration

Internal CO₂ concentration can be expressed as

$$C_i = C_s - \frac{A}{g_s} \quad (69)$$

Leaf surface CO₂ concentration, C_s can be expressed as

$$C_s = C_a - \frac{A}{g_b} \quad (70)$$

where

C_a = Atmosphere's CO₂ concentration (= 360 ppm)

g_b = Conductance across the laminar boundary layer of a leaf (mol m⁻² s⁻¹) for CO₂ exchange

$$g_b = \frac{1}{R_a + R_{b,CO_2}} \quad (71)$$

Both, W_j and W_c take the algebraic form

$$V_c - 0.5V_o = \frac{aC_i - ad}{eC_i + b} \quad (72)$$

CO₂ compensation point in the absence of dark respiration, Γ can be calculated as

$$\Gamma = \frac{0.105 \cdot K_c \cdot [O_2]}{K_o} \quad (73)$$

where

- K_c = Michaelis-Menten coefficients for CO₂ (= 333 microbars at 25 °C)
- K_o = Michaelis-Menten coefficients for O₂ (= 295 millibars at 25 °C)
- $[O_2]$ = Partial pressures of O₂ in the intercellular air space (= 210 μmol/mol)

2.11.5. Derivation of photosynthetic flux density of a leaf

The goal is to derive an equation describing A that is independent of C_s , C_i , and g_s , the term C_s is eliminated by inserting Equation 74 into Equations 73 and 69.

$$C_i = C_a - \frac{A}{g_b} - \frac{A}{g_s} \quad (74)$$

$$g_s = \frac{mArh}{C_a - \frac{A}{g_b}} + b' \quad (75)$$

The term, g_s is eliminated by inserting Equation 79 to Equation 78.

$$\begin{aligned}
 C_i &= C_a - \frac{A}{g_b} - \frac{A}{\frac{mArh}{C_a - \frac{A}{g_b}} + b'} = C_a - \frac{A}{g_b} - \frac{A \left(C_a - \frac{A}{g_b} \right)}{mArh + b' \left(C_a - \frac{A}{g_b} \right)} \\
 &= \frac{C_a g_b \left[mArh + b' \left(C_a - \frac{A}{g_b} \right) \right] - A \left[mArh + b' \left(C_a - \frac{A}{g_b} \right) \right] - A g_b \left(C_a - \frac{A}{g_b} \right)}{g_b \left[mArh + b' \left(C_a - \frac{A}{g_b} \right) \right]} \\
 &= \frac{C_a g_b mArh + C_a^2 b' g_b - C_a b' A - m r h A^2 - A b' C_a + \frac{b' A^2}{g_b} - g_b A C_a + A^2}{g_b mArh + b' C_a g_b - b' A} \\
 &= \frac{\left(1 + \frac{b'}{g_b} - m r h \right) A^2 + C_a (g_b m r h - 2 b' - g_b) A + C_a^2 b' g_b}{(g_b m r h - b') A + b' C_a g_b} = \frac{\alpha A^2 + \beta A + \gamma}{\theta A + \frac{\gamma}{C_a}}
 \end{aligned} \tag{76}$$

$$\alpha = 1 + \frac{b'}{g_b} - m r h \tag{77}$$

$$\beta = C_a (g_b m r h - 2 b' - g_b) \tag{78}$$

$$\gamma = C_a^2 b' g_b \tag{79}$$

$$\theta = g_b m r h - b' \tag{80}$$

From Equations 70 and 76,

$$\begin{aligned}
 A + R_d &= \frac{a C_i - a d}{e C_i + b} \\
 A e C_i + A b + R_d e C_i + R_d b &= a C_i - a d \\
 (A e + R_d e - a) C_i &= -a d - R_d b - A b \\
 C_i &= \frac{-a d - R_d b - A b}{A e + R_d e - a}
 \end{aligned} \tag{81}$$

With Equations 80 and 85,

$$\frac{\alpha A^2 + \beta A + \gamma}{\theta A + \frac{\gamma}{c_a}} = \frac{-ad - R_d b - Ab}{Ae + R_d e - a}$$

$$(\alpha A^2 + \beta A + \gamma)(Ae + R_d e - a) = (-ad - R_d b - Ab) \left(\theta A + \frac{\gamma}{c_a} \right)$$

$$\begin{aligned} \therefore e\alpha A^3 + \alpha R_d e A^2 - a\alpha A^2 + e\beta A^2 + \beta R_d e A - a\beta A + e\gamma A + R_d e\gamma - a\gamma = \\ -ad\theta A - ad\frac{\gamma}{c_a} - R_d b\theta A - R_d b\frac{\gamma}{c_a} - b\theta A^2 - b\frac{\gamma}{c_a} A \end{aligned}$$

$$\begin{aligned} \therefore e\alpha A^3 + (e\beta + b\theta - a\alpha + \alpha R_d e)A^2 + \left(e\gamma + b\frac{\gamma}{c_a} - a\beta + ad\theta + \beta R_d e + \right. \\ \left. R_d b\theta \right) A - a\gamma + ad\frac{\gamma}{c_a} + R_d e\gamma + R_d b\frac{\gamma}{c_a} = 0 \end{aligned}$$

$$\begin{aligned} \therefore A^3 + \frac{e\beta + b\theta - a\alpha + \alpha R_d e}{e\alpha} A^2 + \frac{e\gamma + b\frac{\gamma}{c_a} - a\beta + ad\theta + \beta R_d e + R_d b\theta}{e\alpha} A \\ + \frac{-a\gamma + ad\frac{\gamma}{c_a} + R_d e\gamma + R_d b\frac{\gamma}{c_a}}{e\alpha} = 0 \end{aligned} \quad (82)$$

The solution of the cubic equation is taken from Press et al. (1989). If Equation 86 is manipulated into the form

$$x^3 + px^2 + qx + r = 0 \quad (83)$$

where

$$\begin{aligned} p &= \frac{e\beta + b\theta - a\alpha + e\alpha R_d}{e\alpha} \\ q &= \frac{e\gamma + b\frac{\gamma}{c_a} - a\beta + ad\theta + eR_d\beta + b\theta R_d}{e\alpha} \\ r &= \frac{-a\gamma + ad\frac{\gamma}{c_a} + e\gamma R_d + R_d b\frac{\gamma}{c_a}}{e\alpha} \end{aligned} \quad (84)$$

three roots for the cubic equation are

$$x_1 = -2\sqrt{Q}\cos\left(\frac{\theta}{3}\right) - \frac{p}{3} \quad (85)$$

$$x_2 = -2\sqrt{Q}\cos\left(\frac{\theta+2\pi}{3}\right) - \frac{p}{3} \quad (86)$$

$$x_3 = -2\sqrt{Q}\cos\left(\frac{\theta+4\pi}{3}\right) - \frac{p}{3} \quad (87)$$

where

$$Q = \frac{p^2-3q}{9} \quad (88)$$

$$R = \frac{2p^3-9pq+27r}{54} \quad (89)$$

$$\theta = \arccos\left(\frac{R}{\sqrt{Q^3}}\right) \quad (90)$$

The photosynthetic flux density of a leaf (A) corresponds to root number three (x_3). The variables a , b , d , and e are coefficient from Equation 76. If W_c is minimal, these coefficients correspond to

$$W_c = V_c - 0.5V_o = \frac{aC_i-ad}{eC_i+b} = \frac{V_{c\max}(C_i-\Gamma)}{C_i+K_c\left(1+\frac{[O_2]}{K_o}\right)} \quad (91)$$

where

- V_{cmax} = Maximum carboxylation rate when RuBP carboxylase/oxygenase is saturated
- K_c = Michaelis-Menten coefficients for CO_2 (= 333 microbars at 25 °C)
- K_o = Michaelis-Menten coefficients for O_2 (= 295 millibars at 25 °C)
- Γ = CO_2 compensation point in the absence of dark respiration
- $[O_2]$ = Partial pressures of O_2 in the intercellular air space (= 210 $\mu\text{mol/mol}$)

$$V_{c \max} = \frac{V_c(25^\circ\text{C}) \exp\left[\frac{(T-298)E}{298RT}\right]}{1 + \exp\left(\frac{ST-H}{RT}\right)} \quad (92)$$

S = constants for Boltzmann distribution temperature function (=710)

H = constants for Boltzmann distribution temperature function (=220,000)

The temperature dependencies of the K_c and K_o can be compensated for with Arrhenius Equation (Farquhar et al. 1980);

$$K_c = K_c(25^\circ\text{C}) \exp\left[\frac{(T-298)E}{298RT}\right] \quad (93)$$

$$K_o = K_o(25^\circ\text{C}) \exp\left[\frac{(T-298)E}{298RT}\right] \quad (94)$$

where

E = Relevant activation energy

R = Universal gas constant

T = Absolute leaf temperature

If W_j is minimal, a , b , d , and e coefficients correspond to

$$W_j = V_c - 0.5V_o = \frac{aC_i - ad}{eC_i + b} = \frac{J(C_i - \Gamma)}{4C_i + 8\Gamma} \quad (95)$$

where J is the potential rate of electron transport and expressed as (Harley et al. 1992):
for sunlit leaves in the j^{th} layer of the canopy

$$J = \frac{\alpha PAR_{sun,j}}{\sqrt{1 + \frac{\alpha^2 PAR_{sun,j}^2}{J_{max}^2}}} \quad (96)$$

and, for shaded leaves in the j^{th} layer of the canopy

$$J = \frac{\alpha PAR_{shade,j}}{\sqrt{1 + \frac{\alpha^2 PAR_{shade,j}^2}{J_{max}^2}}} \quad (97)$$

where

- α = Efficiency of light energy conversion on an incident light basis (= 0.22 mol electrons/mol photons)
- $PAR_{sun,j}$ = Flux density of PAR on sunlit leaves in the j^{th} layer of canopy
- $PAR_{shade,j}$ = Flux density of PAR on shaded leaves in the j^{th} layer of canopy
- J_{max} = Light-saturated rate of electron transport (= 171 at 25 °C for trees)

The temperature dependencies of the J_{max} can be compensated for with Arrhenius Equation (Farquhar et al. 1980);

$$J_{max} = \frac{J_{max}(25^{\circ}\text{C}) \exp\left[\frac{(T-298)E}{298RT}\right]}{1 + \exp\left(\frac{ST-H}{RT}\right)} \quad (98)$$

where

- E = Relevant activation energy
- R = Universal gas constant
- T = Absolute leaf temperature

Table 4 summarizes CO₂ exchange and stomatal conductance model parameters on sunlit/shaded leaves.

Table 4 Summary of parameter values

Variable	Unit	Value
V_{cmax} (25°C)	$\mu\text{mol CO}_2 \text{ m}^{-2}\text{s}^{-1}$	90
J_{max} (25°C)	$\mu\text{mol (e}^-) \text{ m}^{-2}\text{s}^{-1}$	171
K_c (25°C)	Pa	33.3
K_o (25°C)	kPa	29.5
Activation energy value for V_{cmax}	J mol^{-1}	64,637
Activation energy value for J_{max}	J mol^{-1}	37,000
Activation energy value for K_c	J mol^{-1}	65,120
Activation energy value for K_o	J mol^{-1}	13,990
Activation energy value for R_d	J mol^{-1}	51,176
α	$\text{mol e}^- (\text{mol quanta})^{-1}$	0.22
m		10
b'	$\text{mol m}^{-2} \text{ s}^{-1}$	0.02

2.11.6. Weighted stomatal conductance for each layer

According to Norman (1982), stomatal conductance for sunlit and shaded leaves in a layer of the canopy can be weighted with sunlit and shaded leaf area index for that layer to estimate the stomatal conductance for that layer.

$$g_{s,j} = \Delta F_j^* g_{s,sun,j} + (\Delta F - \Delta F_j^*) g_{s,shade,j} \quad (99)$$

2.11.7. Stomatal conductance for whole layers

Stomatal conductance for the whole layers can be calculating by taking summation of the stomatal conductance for each layer. Stomatal resistance is a reciprocal of stomatal conductance.

$$g_s = \sum_{j=1}^N g_{s,j} \quad (100)$$

2.12. Transpiration flux calculation

Transpiration is the escape of water vapor from plants as controlled to a considerable degree by leaf resistances. The process is comprised of two stages: evaporation of water from cell walls and diffusion out of the leaf mainly through stomata (Kramer 1983). Transpiration flux, T_f ($\text{g m}^{-2} \text{hr}^{-1}$) is estimated as (Kramer 1983):

$$T_f = \frac{C_{leaf} - C_{air}}{\frac{1}{g_s} + R_a} \cdot \frac{3600}{LAI} \quad (101)$$

where

- C_{leaf} = Water vapor concentration at the evaporating surfaces within the leaf (g m^{-3})
- C_{air} = Water vapor concentration in the air (g m^{-3})
- g_s = Stomatal conductance (s m^{-1})
- R_a = Aerodynamic resistance (s m^{-1})
- LAI = Leaf area index

C_{leaf} and C_{air} are calculated as (Monteith and Unsworth 1990):

$$C_{leaf} = \frac{M_w e_s}{RT} = \frac{2165 e_s}{T} \quad (102)$$

$$C_{air} = \frac{M_w e}{RT} = \frac{2165e}{T} \quad (103)$$

- M_w = Molecular weight of water (=18 g mol⁻¹=18000 g kmol⁻¹)
 R = Universal gas constant (=8.314 J mol⁻¹ K⁻¹)=8.314 kPa m⁻³ kmol⁻¹ K⁻¹)
 e_s = Saturation vapor pressure (kPa)
 e = Vapor pressure (kPa)
 T = Temperature (K)

The hourly transpiration flux mass per unit canopy cover, T_f (g m⁻² hr⁻¹) is converted to depth (m hr⁻¹) by multiplying 10⁻⁶ (1 g of water flux m⁻² = 10⁻⁶ ton m⁻² = 10⁻⁶ m³ m⁻² = 10⁻⁶ m). It is then adjusted based on hourly potential evapotranspiration (PET) (m hr⁻¹) from inside of trees and the ground calculated by the weather pre-processor (Hirabayashi and Endreny 2015). When PET is larger than T_f during the leaf-on season, the ratio between T_f and PET is averaged;

$$\bar{R} = \frac{\sum_t(T_{ft}/PET_t)}{n} \quad (104)$$

For T_f during the leaf-off season as well as when T_f exceeds PET throughout a year, T_f is calculated by;

$$T_f = \bar{R} \cdot PET \quad (105)$$

2.13. Air quality improvement calculation

Hourly air quality improvement per unit tree cover due to the dry deposition of air pollutants, I_{unit} (%) is calculated as:

$$I_{unit} = \frac{F}{F+M_{total}} \cdot 100 \quad (106)$$

where

- F = Pollutant flux (g m⁻² h⁻¹)
 M_{total} = Total air pollutant mass per unit tree cover (g m⁻² h⁻¹)

$$M_{total} = H \cdot C \quad (107)$$

where

- H = Urban mixing height (m)
 C = Air pollutant concentration ($\text{g m}^{-3} \text{ h}^{-1}$)

For PM_{2.5}, if the net flux (F) is negative

$$I_{unit} = \frac{F}{M_{total}} \cdot 100 \quad (108)$$

Hourly air quality improvement for total tree cover, I_{total} (%) is calculated as:

$$I_{total} = \frac{F \cdot \frac{T_c}{100}}{F \cdot \frac{T_c}{100} + M_{total}} \cdot 100 \quad (109)$$

where

- T_c = Total tree cover in the city (%)

For PM_{2.5}, if $F = -M_{total}$ and $T_c = 100$, the denominator in equation 111 becomes 0 and I_{total} becomes infinity. To avoid this, the equation below should be used.

$$I_{total} = I_{unit} \cdot \frac{T_c}{100} \quad (110)$$

2.14. Air pollutant concentration change calculation

Change in air pollutant concentration can be calculated as

$$\Delta C = \frac{C}{1 - \frac{I_{total}}{100}} - C \quad (111)$$

where

- ΔC = Air pollutant concentration change (ppm for CO, NO₂, SO₂, and O₃ and $\mu\text{g m}^{-3}$ for PM₁₀ and PM_{2.5})
 C = Air pollutant concentration (ppm for CO, NO₂, SO₂, and O₃ and $\mu\text{g m}^{-3}$ for PM₁₀ and PM_{2.5})

2.15. Monetary value calculation

Monetary value of pollution removal by trees is estimated using the median externality values for the United States for each pollutant (Murray et al. 1994, Ottinger et al. 1990) adjusted to 2007 dollars based on the producer price index (US Dept. of Labor 2008). The externality values are: CO=\$1,407t⁻¹, NO₂=\$9,906t⁻¹, PM10=\$6,614t⁻¹, SO₂=\$2,425t⁻¹. Externality value for O₃ was set to equal the value for NO₂.

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