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Power function decay of hydraulic conductivity for a TOPMODEL-based infiltration routine

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Abstract:

TOPMODEL rainfall-runoff hydrologic concepts are based on soil saturation processes, where soil controls on hydrograph recession have been represented by linear, exponential, and power function decay with soil depth. Although these decay formulations have been incorporated into baseflow decay and topographic index computations, only the linear and exponential forms have been incorporated into infiltration subroutines. This study develops a power function formulation of the Green and Ampt infiltration equation for the case where the power $n = 1$ and 2. This new function was created to represent field measurements in the New York City, USA, Ward Pound Ridge drinking water supply area, and provide support for similar sites reported by other researchers. Derivation of the power-function-based Green and Ampt model begins with the Green and Ampt formulation used by Beven in deriving an exponential decay model. Differences between the linear, exponential, and power function infiltration scenarios are sensitive to the relative difference between rainfall rates and hydraulic conductivity. Using a low-frequency 30 min design storm with 4·8 cm h^{-1} rain, the $n = 2$ power function formulation allows for a faster decay of infiltration and more rapid generation of runoff. Infiltration excess runoff is rare in most forested watersheds, and advantages of the power function infiltration routine may primarily include replication of field-observed processes in urbanized areas and numerical consistency with power function decay of baseflow and topographic index distributions. Equation development is presented within a TOPMODEL-based Ward Pound Ridge rainfall-runoff simulation. Copyright © 2006 John Wiley & Sons, Ltd.

KEY WORDS TOPMODEL; hydrological model; runoff; soil transmissivity; Croton; Ward Pound Ridge

INTRODUCTION

Gravitational and capillary forces govern watershed drainage, and gravity forces in steeper terrain take relatively more control in determining areas of soil saturation, baseflow regimes, and hydrograph response. Beven and Kirkby (1979) distilled these ideas into a set of TOPMODEL concepts describing topographic index (TI) controls on saturation and runoff response, where the TI is a function of contributing area per contour width a and local slope angle β: $\text{TI} = \ln(a/\beta)$.

TOPMODEL was initially constrained by assumptions of a topographically controlled watershed with shallow depth to bedrock, uniform rainfall, saturation excess runoff, and exponential decay of hydraulic conductivity with soil depth. A review of many TOPMODEL studies (Beven *et al.*, 1995) indicated that the TI concepts often succeeded in describing variable source area and runoff dynamics, yet the simplified TI did sometimes fail. Model simplicity in governing assumptions, Beven (1997b) argued, was a likely cause for occasional model failure and reason for modification to suit local hydrologic mechanisms. Visualization of TOPMODEL predictions across the watershed, followed by comparisons with observations, facilitates consideration of fundamental modifications (Ambroise *et al.*, 1996; Beven, 1997b). A subsequent review of TOPMODEL applications (Beven, 1997a) illustrated several independent modifications of TOPMODEL

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equations, well beyond parameter value adjustments, which better reflected the hydrologic response of particular systems. These applications introduced alternative TOPMODEL assumptions, including power function or linear decay of hydraulic conductivity, a soil hydraulic conductivity weighted TI, infiltration excess runoff, and multiple watersheds with varying rainfall and soil depths.

This paper reports on development of power function decay into Green–Ampt soil infiltration equations to provide an alternative to exponential decay and provide numerical consistency with the power function distribution of TI and baseflow. Although several TOPMODEL applications have included power function decay into TI and baseflow computations (Ambroise *et al.*, 1996; Duan and Miller, 1997; Iorgulescu and Musy, 1998), none has presented adjustments to infiltration equations. Application of this work occurs within the object-oriented topographic model OBJTOP, a new addition to the TOPMODEL library (Wang *et al.*, 2005a,b).

POWER FUNCTION TRANSMISSIVITY PROFILES

Ambroise *et al.* (1996) report that two functions that best fit hydrograph recession curves of baseflow Q_b across time t are (1) the exponential function for small streams in deep soils

$$Q_b = Q_s\ e^{-t/t_s} \tag{1}$$

and (2) the second-order hyperbolic function for streams in relatively shallow soils

$$Q_b = Q_s(-t/t_s)^{-2} \tag{2}$$

where Q_s is a reference discharge and t_s is a scaling time. TOPMODEL was originally structured to provide a first-order hyperbolic function for recession:

$$Q_b = Q_s(-t/t_s)^{-1} \tag{3}$$

and failed to match the second-order parabolic recession in France's Ringelbach catchment. Equation (3) originated from an exponential decay function for transmissivity T_z with depth z as a function of surface transmissivity T_0, as established by Beven (1982).

$$T_z = T_0\ e^{-S_i/m} \tag{4}$$

where S_i is the local soil moisture storage deficit and m is a scaling parameter describing the change of T_z with depth z. The exponential behaviour of transmissivity, or hydraulic conductivity K_z, was put forward on the basis of experimental tests as a reasonable description for a large variety of soils.

Ambroise *et al.* (1996), however, illustrated that a downslope transmissivity profile described by a parabolic function, rather than the original TOPMODEL exponential decay function, resulted in the desired second-order parabolic recession curve. The equation was given as

$$T_z = T_0(1 - S/m)^2 \tag{5}$$

This formulation restricts m to an upper limit of the maximum storage deficit. Duan and Miller (1997) generalized the parabolic transmissivity function presented by Ambroise *et al.* (1996) to create a power function for local subsurface transmissivity:

$$T_z = T_0[1 - S/(mn)]^n \tag{6}$$

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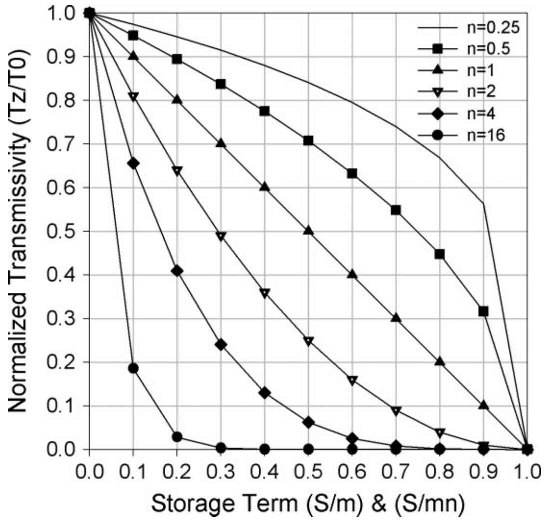

Figure 1. Trend in the normalized transmissivity ratio T_z/T_0 for Equations (6) and (7) using a range of values of soil moisture deficit *S*, scaling parameter *m*, and the power function *n*

Equation (6) requires *S* be less than or equal to the product *mn*. Iorgulescu and Musy (1998) also generalized the Ambroise *et al*. (1996) parabolic transmissivity function into a power law transmissivity profile without incorporating *n* into the quotient, given as

$$T_z = T_0(1 - S/m)^n \tag{7}$$

Figure 1 shows normalized transmissivity, or T_z/T_0, response for Equations (6) and (7), for sets of right-hand-side storage deficit ratio values between zero and one, with *n* values set between 0·25 and 16. Note how T_z becomes a smaller fraction of T_0 as the storage deficit ratio, *S/m* and *S*/(*mn*), increases and as the power function decay rate *n* increases. Keeping the right-hand side equal for both equations requires that *S* and/or *m* change for a given value of *n*, and parameters cannot be simply exchanged between formulations.

Infiltration equations were not derived in the earlier power function decay work (Ambroise *et al*., 1996; Duan and Miller, 1997; Iorgulescu and Musy, 1998), which focused instead on derivations for TOPMODEL TI and baseflow equations. This paper presents power function infiltration equations, incorporated into the OBJTOP model, as an alternative to the exponential function infiltration equations derived by Beven (1984). Green and Ampt (1911) infiltration theory, as used in Beven (1984), is also used in this power function application. It is important to note that exponential decay infiltration equations, as presented by Beven (1984), remain an important simulation scheme option. The generalized power function profile of Iorgulescu and Musy (1998), which is a simpler formulation, was used in this work for derivation of the power function infiltration equation. Likewise, the choice of power function simulation necessitated incorporation of the Iorgulescu and Musy (1998) power-function-adjusted TI and subsurface flow formulations, given as

$$\mathrm{TI} = (a/\tan\ \beta)^{1/n} \tag{8}$$

$$\overline{q}_{\mathrm{subsurface}} = T_0\lambda^{-n}(1 - \overline{s}/m)^n \tag{9}$$

where $\lambda = 1/A\int(a/\tan\beta)^{1/n}\,\mathrm{d}A$ is the average TI, *A* is the total catchment area, and $\overline{s} = m[1 - (R/T_0)^{1/n}\lambda]$ is the average soil moisture deficit under λ, where *R* is the recharge rate. Linear (where $n = 1$) and parabolic (where $n = 2$) are special cases of the above generalized Equations (8) and (9) presented by Iorgulescu and Musy (1998).

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POWER FUNCTION INFILTRATION EQUATIONS

A combined infiltration and saturation excess mechanism is used in this work, where the processes simulation starts with a check for infiltration excess runoff if rain rates are greater than infiltration rates and infiltrates ponded or lower rain-rate water given available storage. Infiltrated water becomes the input of the saturation excess runoff process, which starts when the water-table meets the surface. Adjustments to storage occur by a spatial resorting of the saturation based on the TI, subsurface flow drainage, and losses to evapotranspiration.

The power function infiltration equations presented in this section follow from earlier work of Beven (1984) with Green and Ampt infiltration presented for an exponential function. In this work, infiltration-excess overland flow begins with an infiltration rate i, identified by Beven (1984) as

$$i = \frac{\mathrm{d}I}{\mathrm{d}t} = \frac{\Delta\psi + z}{\displaystyle\int_{z=0}^{z=z} \frac{\mathrm{d}z}{K_z}} \tag{10}$$

in which I is the cumulative infiltration and $\Delta\psi$ is effective wetting front suction. For exponential decay $K_z = K_0\ \mathrm{e}^{-fz}$, where

$$f = \frac{\Delta\theta}{m} \tag{11}$$

and $\Delta\theta = (\theta_s - \theta_i)$, where θ_s is the saturated volumetric soil moisture content and θ_i is the initial volumetric soil moisture content. θ_s is the saturated volumetric soil moisture content, which is the formulation used in many TOPMODEL applications. For power function decay

$$K_z = K_0(1 - fz)^n \tag{12}$$

This study developed and incorporated two special transmissivity profiles, linear ($n = 1$) and parabolic ($n = 2$), to simulate infiltration excess overland flow.

Linear application

Power function infiltration under case one (where $n = 1$) with substitution of Equation (12) into Equation (10) gives

$$i = \frac{\mathrm{d}I}{\mathrm{d}t} = \frac{K_0(\Delta\psi + z)}{\displaystyle\int_{z=0}^{z=z} (1 - fz)^{-1}\ \mathrm{d}z} \tag{13}$$

We assume that at the ponding time t_p the cumulative infiltration I_p has penetrated as a wetting front to a depth z_p, where

$$z_p = I_p/\Delta\theta \tag{14}$$

Further, if r is defined as the constant infiltration rate before ponding, then I_p is defined as the product of r and t_p, and storage suction factor (Beven, 1984) is given as

$$C = \Delta\psi\Delta\theta \tag{15}$$

Substitution of Equations (14) and (15) into Equation (13) gives at the onset of ponding

$$i = \frac{\mathrm{d}I}{\mathrm{d}t} = \frac{K_0(\Delta\psi + z)}{\displaystyle\int_{z=0}^{z=z_p} (1 - fz)^{-1}\ \mathrm{d}z} = \frac{-fK_0(\Delta\psi + z_p)}{[\ln(1 - fz)]_0^{z_p}} = \frac{-fK_0(\Delta\psi + I_p/\Delta\theta)}{\ln(1 - fz_p)}$$

$$r = \frac{-fK_0(\Delta\psi + I_p/\Delta\theta)}{\ln(1 - fI_p/\Delta\theta)} = \frac{-\dfrac{K_0}{m}(C + I_p)}{\ln(1 - I_p/m)} \tag{16}$$

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After ponding begins, Equation (16) becomes

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \frac{-\dfrac{K_0}{m}(C+I)}{\ln(1-I/m)} \; {-}{-}{>} \int_{I_\mathrm{p}}^{I} \frac{\ln(1-I/m)}{C+I}\mathrm{d}I = -\frac{K_0}{m}(t-t_\mathrm{p}) \tag{17}$$

Implementation of Equation (17) is achieved with the Simpson method for numerical solution to obtain solutions for I at any time t after ponding when t_p and I_p are known from Equation (16). It is set that the maximum cumulative infiltration I should be less than m, the maximum storage deficit.

Parabolic application

Power function infiltration under case two, where $n = 2$ in Equation (12), gives

$$K_z = K_0(1-fz)^2 \tag{18}$$

Substituting Equation (18) into Equation (10) gives

$$i = \frac{\mathrm{d}I}{\mathrm{d}t} = \frac{\Delta\psi + z}{\displaystyle\int_{z=0}^{z=z} \frac{\mathrm{d}z}{K_z}} = \frac{\Delta\psi + z}{\displaystyle\int_{z=0}^{z=z} \frac{\mathrm{d}z}{K_0(1-fz)^2}} = \frac{-fK_0(\Delta\psi + z)}{\displaystyle\int_{z=0}^{z=z} (1-fz)^{-2}\,\mathrm{d}z} \tag{19}$$

Further, Equations (11), (13) and (14) and the given infiltration rate $i = r$, where $I_\mathrm{p} = rt_\mathrm{p}$, when substituted into Equation (19) gives

$$r = \frac{\mathrm{d}I}{\mathrm{d}t} = \frac{-fK_0(\Delta\psi + z)}{\left[\dfrac{1}{1-fz}\right]_0^z} = \frac{K_0}{I_\mathrm{p}}(C+I_\mathrm{p})(1-I_\mathrm{p}/m) \tag{20}$$

After ponding starts, Equation (20) remains functional in the following form

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \frac{K_0}{I}(C+I)(1-I/m) \; {-}{-}{>} \int_{I_\mathrm{p}}^{I} \frac{I\,\mathrm{d}I}{(C+I)(1-I/m)} = \int_{t_\mathrm{p}}^{t} K_0\,\mathrm{d}t \tag{21}$$

The integrals in Equation (21) reduce to

$$p\ln\left(\frac{C+I}{C+I_\mathrm{p}}\right) + qm\ln\left(\frac{1-I_\mathrm{p}/m}{1-I/m}\right) = K_0(t-t_\mathrm{p}) \tag{22}$$

in which the p and q constants are defined as

$$p = -Cm/(m+C)$$

and

$$q = m/(m+C)$$

The Simpson numerical method is used to solve Equation (22) for I at any time t after ponding when t_p and I_p are known from Equation (20).

Differences between the power function $n = 1$ and $n = 2$ infiltration rates become apparent at ponding, and otherwise infiltration equals precipitation. Figure 2 shows the decaying infiltration rate for a 5 min time-step simulation of 30 min of intense precipitation at 0·048 m h^{-1} on a sandy loam soil with saturated hydraulic conductivity of 0·011 m h^{-1}, matric suction of 0·11 m, and porosity of 0·45. In this scenario, the $n = 2$ formulation provides a more rapid decay of infiltration and a quicker generation of surface runoff than the exponential or $n = 1$ formulations. Such simulation flexibility may be important for urban studies with

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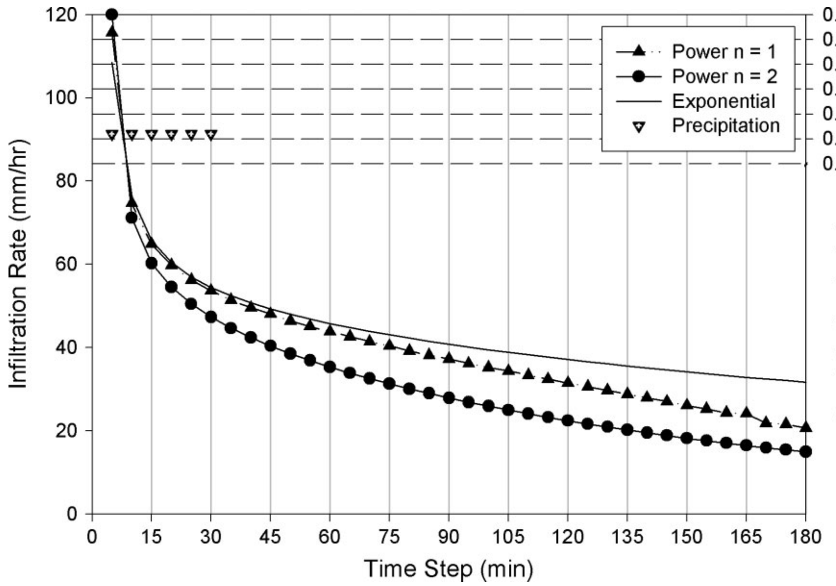

Figure 2. Infiltration rates as time-series for the two different power function formulations ($n = 1$, $n = 2$) and one exponential function formulation given precipitation of 0·24 m h^{-1} in a model test simulation

increased distribution of infiltration excess runoff. Isolated infiltration simulations were performed prior to running the full watershed model (which can run at any user-defined time-step) to test the functionality and distinct decay differences between the power function and exponential function infiltration equations.

Differences between the two power function infiltration scenarios are insignificant for rainfall rates less than hydraulic conductivity; however, the power function infiltration contribution remains useful. It provides an internally consistent conceptual framework to couple with TI and baseflow power function methods, and it represents the often-observed power function decay of conductivity. This new power function algorithm has been incorporated into the object-oriented TOPMODEL-based watershed simulation tool. Testing of the full model, which included power function infiltration, baseflow, and TI distributions, was performed in a small glacially formed watershed of New York, USA, as described below.

STUDY SITE AND FUNCTION TESTING

The power function decay of hydraulic conductivity, developed above, was incorporated into the object-oriented TOPMODEL-based watershed simulation tool called OBJTOP (Wang *et al*., 2005a,b). Testing of the full model was performed in the 0·376 km^2 area Ward Pound Ridge (WPR) in Westchester County Park, New York, USA. The WPR watershed was part of a programme to investigate terrestrial flow control processes for the New York City (NYC), USA, Croton drinking water supply area (see Figure 3). Geologic material in this area contains igneous and metamorphic bedrock with upland soils of glacial till with variable thickness, and valley soils of alluvium, peat-muck, and glacial outwash (Heisig, 2000). Site assessment revealed a power function decay of hydraulic conductivity with soil depth. Sampling of saturated hydraulic conductivity K_{sat} was performed using an amoozemeter (Amoozegar, 1992) at the monitoring clusters. Decay of K_{sat} with depth is shown as four data points in Figure 4; and a power function, rather than an exponential function, provided the best approximation of the decay. Digital elevation model data with horizontal spatial resolution of 2 m were obtained from low-elevation aerial photography and were processed with the Quinn *et al*. (1991) multiple flow algorithm for generation of the WPR TI.

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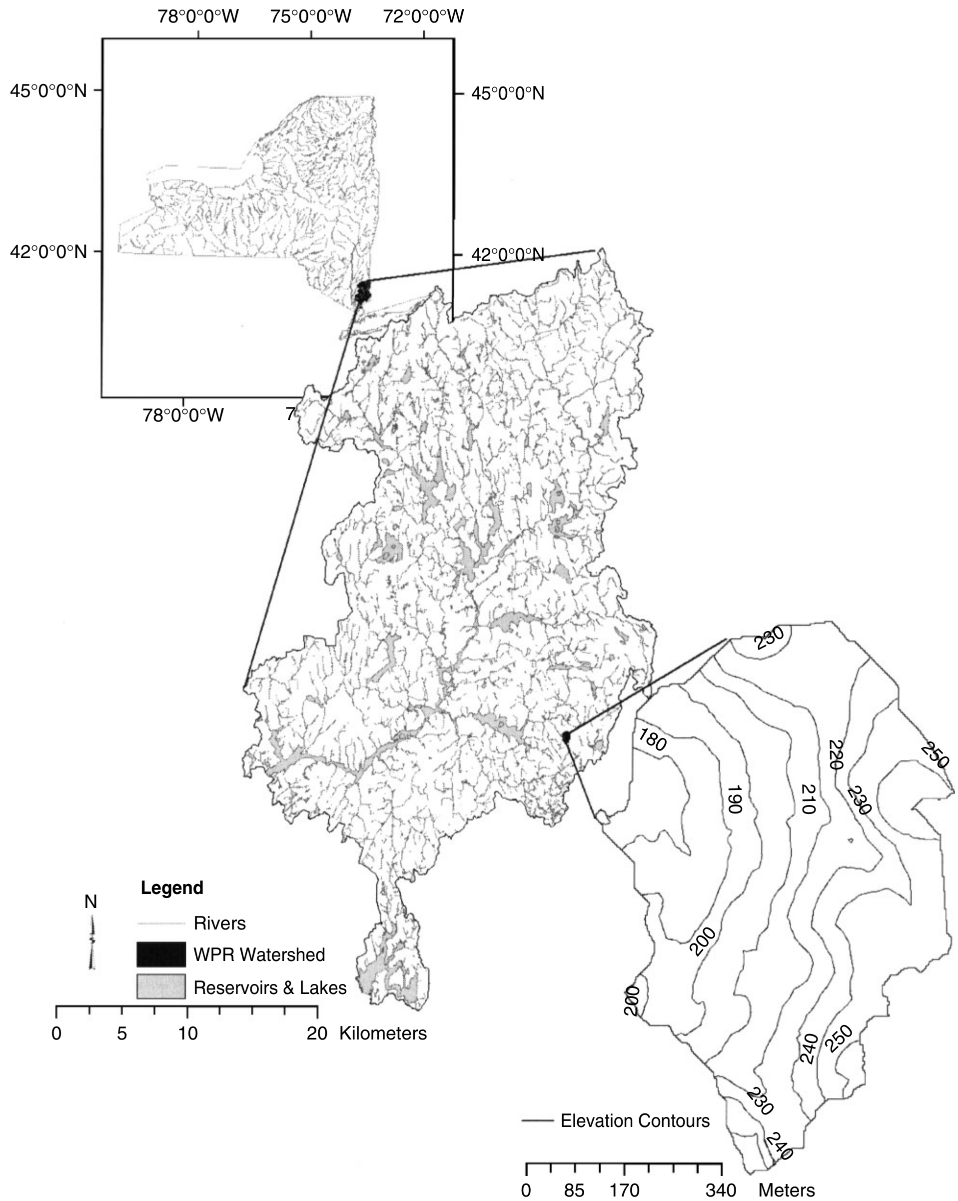

Figure 3. Map of the WPR watershed shown relative to its position in the nearly 1000 km^2 Croton basin, located in southeastern New York State (NYS). The WPR watershed shows elevation contours, in metres, and the Croton and NYS maps show rivers, lakes, and reservoirs

First, OBJTOP was run at an hourly time-step and manually calibrated to WPR hourly discharge for the period of 1 October to 31 December 2001, resulting in the parameter values shown in Table I. NYC's need to close this monitoring programme shortened the calibration period. This exercise was to demonstrate that the complete model could incorporate the power function infiltration methodology, in place of the standard exponential decay approach. A scatter plot fit, with an $r^2 = 0{\cdot}80$, was obtained for discharge in the combined calibration and validation period through 4 March 2002, as shown in Figure 5a. During an intense 25 November 2001 storm event, approximately 15% of runoff was attributed to infiltration excess,

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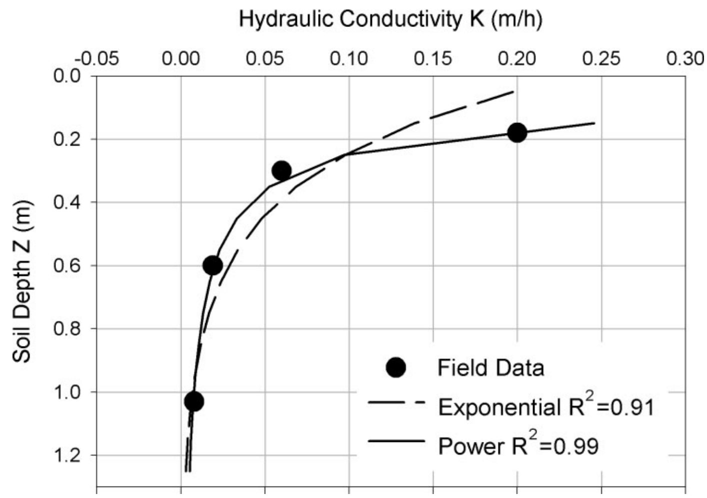

Figure 4. Measured WPR soil hydraulic conductivity with soil depth along with an overlay of power function fit and exponential fit for this decay

Table I. OBJTOP simulation parameters and values for simulation of WPR, New York watershed from October 2001 to March 2002

Parameter name	Parameter value	Descriptions and comments
n	2·0	Exponent of power function decay
m	0·21	A scaling parameter
T_0 (m^2 h^{-1})	0·074	Saturated surface soil transmissivity
T_d (h)	20	Unsaturated zone time delay
MRZD (m)	0·052	Maximum root-zone storage deficit
Q_0 (m h^{-1})	2.6×10^{-6}	Initial discharge
RZD_0 (m)	0·000 01	Initial root-zone storage deficit
ICRV (m h^{-1})	590	Internal channel routing velocity
ψ (m)	0·05	Wetting front suction factor
θ (%)	0·4	Wetted soil moisture content

about 45% to saturation excess, and the remainder delivered as subsurface flow. Only 20% of the soils had conductivity values capable of infiltration excess runoff for rain rate inputs. Calibration obtained a Nash and Sutcliffe (1970) efficiency value of 0·92, which was similar to the value obtained in the exponential decay approach, but more accurately represented the observed soil infiltration processes. Second, predicted discharges from parabolic-power-function- and exponential-function-based infiltration mechanisms were compared for the same simulation period, and are reported in Figure 5b. This regression had an $r^2 = 0.97$, and differences in predicted WPR discharge for these two schemes stem mostly from different variably saturated areas and hydrograph rising limbs. In developed residential areas, with compacted soils, the OBJTOP code provides a variety of infiltration decay methods to increase flexibility in matching the increased incidence of infiltration excess runoff.

CONCLUSIONS

Rainfall-runoff research has observed and simulated power function decay of hydrograph recession curves, and has used power function decay to compute TIs of soil saturation for TOPMODEL studies. Field research

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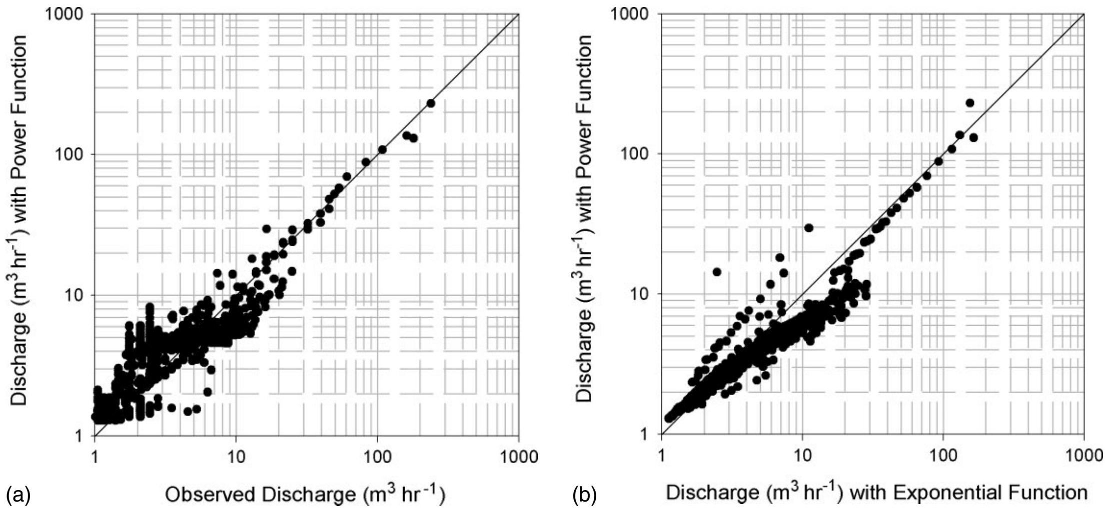

Figure 5. (a) Scatter plot of observed and power function decay simulated discharge from 1 October 2001 to 4 March 2002 in WPR watershed, with a 0·80 coefficient of determination. (b) Scatter plot of exponential decay and power function decay simulated discharge from 1 October 2001 to 4 March 2002 in WPR watershed, with a 0·92 coefficient of determination

in the WPR watershed in the NYC drinking water supply area revealed soil hydraulic conductivity values that had power function decay with soil depth. This paper presents two power function formulations, $n = 1$ and 2, of the Green and Ampt infiltration equation developed to assist simulation in the WPR. The new infiltration routine complements the Green and Ampt exponential decay routine equation derived by Beven for TOPMODEL. In power function infiltration testing, we illustrate how that formulation provides a more rapid decay of infiltration than the exponential form, given a low-frequency, high-intensity 30 min storm raining at 4·8 cm h^{-1}. Such simulations may help capture infiltration excess runoff dynamics, rare in forested areas, but noted in compacted residential sites. Power function simulation of infiltration was provided by the new routine within an object-oriented TOPMODEL code, called OBJTOP. OBJTOP couples power function simulation of infiltration, baseflow transmissivity, and TIs, and brings conceptual consistency to model representation of power-function-based hydrological processes.

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